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Abstract: This document presents an empirical analysis of the *Fitness-based Area-Under-Curve - Bandit* (*F-AUC-Bandit*), an adaptive strategy (or operator) selection method recently proposed in the context of Genetic Algorithms. It is here used to select, while solving the problem, the strategy to be applied for the next offspring generation based on the recent known performance of each of the available ones, within a Differential Evolution algorithm applied to continuous optimization problems. Experimental results are obtained on a testbed of single-objective noiseless functions. The performance gain achieved by the use of adaptive strategy selection methods is shown by comparing *F-AUC-Bandit* with what would be the common naïve choices: the use of a single strategy or the uniform selection between a sub-set of available strategies. *F-AUC-Bandit* is also compared to previously proposed adaptive schemes, showing a significantly better performance (w.r.t. expected running time to achieve a target solution) on most of the functions, while presenting a robust hyper-parameter setting. Although still being not competitive with state-of-the-art continuous optimizers such as the CMA-ES (to which an empirical comparison is also presented), a big enhancement is achieved over the use of the basic Differential Evolution, while also improving over both naïve and existent adaptive methods.

Key-words: Adaptive Strategy Selection, Differential Evolution, Multi-Armed Bandits, ROC Area Under Curve

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Analyse de sélection adaptative de stratégies appliquée à l'algorithme à evolution différentielle sur le banc d'essai de fonctions non-bruitées BBOB 2010

Résumé : Ce document présente une analyse empirique de la *Fitness-based Area-Under-Curve - Bandit* (*F-AUC-Bandit*), une méthode de sélection adaptative de stratégies récemment proposé dans le contexte des algorithmes génétiques. Cette méthode est utilisé ici pour sélectionner, pendant la résolution du problème, la stratégie à être utilisé par l'algorithme à évolution différentielle pour générer la prochaine solution, basée sur les performances récentes des stratégies disponibles. Des résultats expérimentaux sur un banc d'essai de fonctions tests non-bruitées sont présentés. Le gain de performance atteindre par l'utilisation de techniques adaptatives est montré par la comparaison entre *F-AUC-Bandit* et les choix normalement prises par un utilisateur naïf: l'utilisation de seulement une stratégie ou la sélection aléatoire à partir d'un sous-ensemble de stratégies. *F-AUC-Bandit* est comparé aussi avec d'autres techniques adaptatives existantes, en montrant une performance significativement meilleure (par rapport au temps d'exécution espéré pour atteindre une valeur cible) dans la majorité des fonctions, et en présentant en même temps une configuration très robuste pour ces hyper-paramètres. Même si cette combinaison ne peut pas encore rivaliser avec un algorithme d'optimisation état de l'art comme le CMA-ES (avec lequel une comparaison empirique est aussi présenté), un grand gain de performance a été atteint par rapport au algorithme de base, celui à l'évolution différentielle, et aussi par rapport aux choix naïf et adaptatives existantes.

Mots-clés : Sélection adaptative de stratégie, algorithme à évolution différentielle, bandit manchot, aire sous la courbe ROC

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1 Introduction

Differential Evolution (DE) is a very popular evolutionary algorithm. This is mainly because of its simple structure, ease of use, robustness and speed. Because of this, DE has been applied on many real-world applications, such as pattern recognition, neural network training, data mining [1, 8, 24, 9, 6].

However, one of the features that helps making it robust with relation to so many different situations —the number of available strategies for offspring generation [26, 24]— is also responsible for adding an extra difficulty to its use, the definition of which of the available strategies should be applied to the problem at hand. Such choice is problem-dependent, and very sensitive in terms of algorithm performance, what turns to be a non-trivial decision for the user.

An off-line tuning procedure might be used to find the best strategy for the problem at hand. But, besides being computationally expensive, its result (the best single strategy) will always lead to sub-optimal behavior, as exploration tends to be more important in the beginning of the optimization process, while exploitation should be preferred when approaching to the optimum. In other words, different strategies should be applied at different moments of the optimization process, according to its "current needs" in terms of exploration and exploitation.

Defining the way such mixture of strategies will be used during the process becomes yet another optimization problem. This is the main motivation for the use of Adaptive Strategy Selection (*AdapSS*) techniques: based on the recent performance of each strategy on the current optimization process, the strategy to be used on the generation of the next offspring is automatically chosen, while solving the problem.

A new comparison-based technique, the *Fitness-based Area-Under-Curve - Bandit* (*F-AUC-Bandit*), has been recently proposed to this aim [13], being originally assessed in the context of adaptive operator selection within Genetic Algorithms (GAs). It uses a multi-armed bandit algorithm to select the strategy to be applied, with the Area Under the ROC Curve paradigm [5] being used to assess the performance of each strategy, based on the ranks of the fitnesses of the generated offspring, what makes it totally invariant with relation to monotonous transformations over the fitness function.

In this report we extend its empirical validation, coupling the *F-AUC-Bandit* with a DE algorithm, and analyzing it on the context of continuous optimization with the BBOB-2010 noiseless benchmarking suite. Such combination is firstly compared with the common naïve choices, *i.e.*, the basic DE using a single strategy, and the DE with strategies uniformly chosen from the set of available ones (referred to as *Uniform-DE*). Besides, it is also compared with previously proposed adaptive schemes, listed as follows:

- the *PM-AdapSS-DE* [16] adaptive strategy selection technique, a method that uses the *Probability Matching* (*PM*) strategy selection and a credit assignment scheme based on the *relative* fitness improvements;
- the *Adaptive Pursuit* (*AP*) [28] adaptive strategy selection technique, here being fed by extreme value based rewards [10];
- the *Dynamic Multi-Armed Bandit* (*DMAB*) [7], also using the extreme value based rewards;

F-AUC-Bandit is also compared with the three other rank-based approaches proposed in the same paper [13], the one that uses a fitness-based sum of ranks as credit assignment scheme (referred to as *F-SR-Bandit*), and the counterparts of *F-AUC-Bandit* and *F-SR-Bandit*, using the rank of the fitness improvements instead of the rank of the real fitness values. *F-AUC-Bandit* was chosen to be the main technique in this report because it achieved overall better results, even if not much significant difference can be found between it and the other rank-based techniques.

Lastly, in order to have an idea about the performance of *F-AUC-Bandit* with relation to state-of-the-art continuous optimizers, it was also compared with the CMA-ES [3]. Although it did not show to be competitive, a better performance could be achieved by simply tuning more carefully the parameters of DE (population size NP, mutation scaling factor F, and crossover rate CR), but this is out of the current scope of this work, which is rather to present yet another proof-of-concept of the *Adaptive Strategy (or Operator) Selection* paradigm.

As preliminaries, the DE algorithm will be briefly overviewed in Section 2. The adaptive strategy selection techniques are briefly described in Section 3 (we refer the reader to the original papers [13, 16, 28, 7, 10] for a more complete view). Section 4 presents the experimental settings and Section 5 describes the hyper-parameter tuning that were used to generate the empirical results, presented in Section 6. A brief conclusion is given in Section 7

2 Differential Evolution

Differential Evolution (DE) [26, 24] is an evolutionary algorithm that uses a differential mutation procedure that consists in the addition of the weighted difference of two or more population vectors into a third one. DE loops over the following steps:

Selection For the generation of each offspring, up to five different individuals (depending on the mutation strategy that will be used) are randomly selected from the population.

Mutation Many different mutation schemes or *strategies* can be considered. We list a few here:

1) "DE/rand/1":
$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \cdot \big(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}\big) \tag{1}$$

2) "DE/rand/2":
$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \cdot \big(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}\big) + F \cdot \big(\mathbf{x}_{r_4} - \mathbf{x}_{r_5}\big) \tag{2}$$

3) "DE/rand-to-best/2":
$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \cdot \big(\mathbf{x}_{best} - \mathbf{x}_{r_1}\big) + F \cdot \big(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}\big) + F \cdot \big(\mathbf{x}_{r_4} - \mathbf{x}_{r_5}\big) \tag{3}$$

4) "DE/current-to-rand/1":
$$\mathbf{v}_i = \mathbf{x}_i + F \cdot \big(\mathbf{x}_{r_1} - \mathbf{x}_i\big) + F \cdot \big(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}\big) \tag{4}$$

where i is in $1, \ldots, NP$ with NP being the population size, $\mathbf{x}_i$ represents the current individual, $\mathbf{x}_{best}$ is the best individual in the current generation, r_1, r_2, r_3, r_4, r_5 are individuals randomly chosen from the population, being $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq i$. F is a parameter, the mutation scaling factor, in the range $]0, 2]$,

Crossover $\vec{u}_i$ is the resulting individual of the crossover between the parent $\vec{x}_i$ and the mutant candidate $\vec{v}_i$ which is generated by choosing component by component between those of $\vec{v}_i$ and $\vec{x}_i$ with probability CR and $1-CR$ respectively (CR being a parameter), with the exception that one random component of $\vec{u}_i$ must correspond to that of $\vec{v}_i$,

Replacement After the creation of an entire new population, the individual $\vec{u}_i$ replaces $\vec{x}_i$ in the next generation population if there is an improvement.

The setting of parameters NP, CR, F and the choice of the strategies is a sensitive issue in the sense that choosing the right strategy and setting the algorithm parameters is very much dependent on the type of problem considered. This specially motivates the use of adaptive strategy selection techniques.

3 Adaptive Strategy Selection

To do *Adaptive Strategy* (or Operator) *Selection*, there is the need of defining how the impact of the application of a given strategy is assessed, *i.e.*, how to reward the strategy after its "production", which is referred to as the *Credit Assignment* mechanism; and based on these assessments, there is the need of defining how to select the next strategy to be applied, which is called the *Strategy Selection* scheme. The *F-AUC-Bandit* is briefly analyzed in the following, as well as the other adaptive schemes used as baseline for comparison, focusing on how they handle these two issues.

3.1 Fitness-AUC Bandit

The *F-AUC-Bandit* algorithm, recently proposed in the context of GAs [13], uses the Area Under the ROC Curve (AUC) paradigm to assess the empirical quality of each strategy. The AUC is a criterion originally used in *Machine Learning* to compare binary classification rules [5]. In the context of *AdapSS*, it shows how good one strategy is, by comparing the rewards received after its recent applications with the rewards received by the others.

Instead of being calculated based on the received raw rewards, in this work the AUC uses the ranks of these rewards, what improves its robustness with relation to different problems (there is no need of re-scaling the algorithm to the different possible ranges of rewards). Besides, by directly using the rank of the fitnesses of the generated offspring, instead of the commonly used fitness improvements, it becomes a total comparison-based method, invariant with relation to monotonous transformations over the fitness function. This is what we refer to as the *Fitness-based AUC* credit assignment scheme. In case the generated offspring does not improve over its parent, a null reward is assigned.

Figure 1, reproduced from [13], illustrates an example computation of the AUC. Briefly, it is the total area upper bounded by the *Receiving Operator Curve* (ROC), represented by the solid line in the example. Computing the quality of a given strategy consists of going down the sorted list of raw rewards, and drawing, starting from the origin, a vertical segment each time the strategy under assessment is found in the list, a horizontal one otherwise, and a diagonal in case of ties.

In this example, for the sake of clarity, each rank position has the same weight on the calculation of the reward, *i.e.*, each segment has the same length than the others, no matter its ranking. But it is clear that the initial rank positions (the best raw rewards) should have a higher impact on this quality estimation. To this aim, a decay factor can be applied. Being W the size of the sliding window that stores the recent raw rewards received by all the strategies, and r the rank position of a given reward, the length of its segment in the ROC curve (*i.e.*, its importance in the AUC computation) can be calculated as $D^r(W - r)$, with $D \in]0, 1]$ being the decay factor that defines how skewed is this ranking distribution. A linear decay is achieved by using $D = 1$; smaller D, faster the decay.

A multi-armed bandit technique, based on the UCB multi-armed bandit formula [2, 13], is then used to select the next strategy to be applied, according to the presented quality estimation. The main difference is that, as there is no much sense in calculating statistics over statistics, in this case the empirical

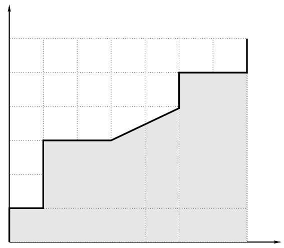

Figure 1: Sample computation of AUC reward: only two operators are involved, and the sorted list contains the operators in the order (1 2 1 1 2 2 [2 2 1] 1 2 2 1), with [2 2 1] meaning that these 3 positions have the same raw reward, leading to the diagonal line between points (3 3) and (5 4) (dotted lines are spaced by 1). In case of decay, the width of the squares would decrease leftward and upward.

quality used by the UCB formula is equal to the last received reward (the AUC computation), instead of being an average of the received rewards, since the AUC efficiently summarizes the up-to-date quality of the strategy with relation to the others, while n refers to the number of times the strategy appears in the current reward sliding window.

Besides the decay factor D, the *F-AUC-Bandit* algorithm requires the definition of a scaling factor C, that is used to balance the importance of its exploration and exploitation terms; and also the credits sliding window size W.

3.2 Fitness-SR Bandit

The **Sum of Ranks** (SR) method, as its name already says, credits the operators with the sum of the ranks of the rewards given after its applications, normalized by the sum of all the rank-values, so that the sum of the credits assigned to all operators sum up to 1 . Being K the number of operators, the operator i is rewarded at time t as follows:

$$SR_{i,t} = \frac{\sum_{op_r=i} D^r(W-r)}{\sum_{r=1}^{W} D^r(W-r)} \tag{5}$$

The strategy selection scheme used is the same used by *F-AUC-Bandit*, based on the adapted version of the UCB multi-armed bandit algorithm.

3.3 AUC-Bandit and SR-Bandit

These techniques are the counterparts of *F-AUC-Bandit* and *F-SR-Bandit* that use the rank of the fitness improvements instead of rank of the real fitness values of the generated offspring for the credit assignment of the strategies. Although not being comparison-based, they are still quite robust with relation to different fitness ranges, as presented in [13].

3.4 PM-AdapSS-DE

The *PM-AdapSS-DE* algorithm is much simpler, although presenting competitive results, as shown in its original paper [16]. The credit assignment scheme awards the strategy with the absolute average of the rewards recently received by it. The reward in this case is the relative fitness improvement, *i.e.*, the improvement achieved by the offspring over its parent, normalized by the fitness of the best-so-far individual.

The *Probability Matching* (*PM*) technique [15] uses this received credit to update the empirical quality estimate it keeps for each strategy, with the weight of the received reward being ruled by a user-defined parameter, the adaptation rate $\alpha \in]0, 1]$. The probability of selection of each strategy is then updated proportionally to its empirical estimate with relation to the others. A minimal probability $p_{min} \in [0, 1]$ might be applied, so that no operator gets "lost" during the process [27]. With this, every time a strategy needs to be applied, it is selected from the set of available ones by a roulette-wheel-like process over these up-to-date probabilities.

3.5 Adaptive Pursuit

Adaptive Pursuit (*AP*) [28] is a strategy selection technique based on probabilities, such as the *PM*. The main difference lies in the update of such probabilities: instead of updating it proportionally to the known quality of each strategy, it implements a *winner-takes-all* scheme, by quickly shifting the probability of the current best strategy towards a maximum value, consequently lowering the rates of the other ones towards a minimal value.

The credit assignment coupled with *AP* in this work is the extreme value based one [10], as it was found to be the best among a set of previously existent ones [11, 12] (although in a totally different context).

3.6 Dynamic Multi-Armed Bandit

Dynamic MAB (*DMAB*) [7] is another strategy selection technique originally proposed in the context of operator selection within Genetic Algorithms. As for the other bandit-based methods previously mentioned, it is also based on the UCB [2] formula; but here the empirical estimate $\hat{q}$ refers to the average of the credits assigned, and n accounts to the number of times the given strategy was applied, as in the original formula, with the scaling factor C handling the balance between the exploration and exploitation terms.

The "dynamic" term here refers to the embedding of a restart strategy onto the original MAB algorithm, which re-initializes the MAB process from scratch based on the Page-Hinkley change-detection statistical test [22], thus allowing it to quickly adapt to the new situation with relation to the performance of the strategies.

As for the *AP*, in this work the extreme value based rewards [10] were used as credit assignment within the *DMAB*.

4 Experimental Settings

We test variants of DE on some continuous optimization problems given by the BBOB 2010 experimental framework [19], which provides a whole experimental set-up for testing continuous optimizers. In particular, BBOB 2010 provides test problems [14, 19] in dimensions from 2 to 40 in the form of function instances:

each of the twenty-four functions of the noiseless testbed has fifteen instances, totalizing 360 problems for each dimension. The experiments were performed following the BBOB guidelines [18], with the maximum number of evaluations being fixed at 10^5 times the dimension.

For the parameters of DE, the population size NP was fixed at 10 times the dimension of the search space, and the mutation scaling factor F was set to 0.5. Although a value around 0.9 is usually advocated for the crossover rate CR, it was chosen to set it to 1.0 for this benchmarking exercise, as in this way the DE becomes invariant with relation to rotation, and entirely dependent on the application of the mutation strategies [20]. Given that the main focus of this work it to further empirically assess the *AdapSS* techniques, instead of competing with the best optimizers, it is true to say that no much attention was deserved to the user-defined parameters of DE, what is left for future work.

Between the four rank-based *AdapSS* techniques proposed in [13], the *F-AUC-Bandit* was the one chosen to be compared in a pair-wise fashion with all the baseline techniques, because it presented overall better results (although not much significant differences can be found). Besides the empirical comparison with the other three rank-based techniques (*AUC-B, SR-B, F-SR-B*), it is also compared with: a standard DE using a single mutation strategy (for each of the strategies presented in Section 2), a variant of DE using uniform strategy selection, and also with the following existent adaptive schemes *PM-AdapSS-DE* [16], *Adaptive Pursuit* [28] and *Dynamic MAB* [7], the two latter being fed by extreme value based rewards [10].

Finally, we also compare the performance of *F-AUC-Bandit* with a state-of-the-art optimizer. The CMA-ES with an Increasing POPulation (IPOP-CMA-ES) size restart strategy [3] was tested on the BBOB 2010 test suite with the same parameter tuning as used in [17].

5 Hyper-Parameters Tuning

For the sake of a fair empirical comparison, the adaptive schemes had their hyper-parameters tuned off-line, by means of the F-Race technique [4], as in [13]. The F-Race eliminates candidate configurations as soon as it is possible to conclude, based on the Friedman's two-way analysis of variances by ranks statistical test being applied at a rate of 95%, that a given candidate configuration will not be the best, thus saving computational resources and time. The first elimination round happens after one run over all instances of a given dimension/function class, being done after every run, up to 10 runs or just one configuration left. The parameter values tried for each technique are summarized in Table 1. We refer the reader to the original papers [16, 28, 7, 13] for a more detailed description of each of the mentioned hyper-parameters.

5.1 On the robustness w.r.t the hyper-parameters

Firstly, to try to have a view of the robustness of each technique with relation to its hyper-parameters, 6 different tuning procedures were performed for each dimension $\in \{5, 20\}$, being them: the tuning considering, independently, each of the 5 function classes; and the tuning considering all the functions. The best

Heuristic	H-P	Range	Comments
All	$\mathcal{W}$	$\{50, 100, 500\}$	Window size
AP, *PM*	p_{min}	$\{0; .05; .1; .2\}$	Min. prob. (P)
AP, *PM*	α	$\{.1, .3, .6, .9\}$	Adaptation rate
AP	β	$\{.1, .3, .6, .9\}$	Learning rate
DMAB(PH)	γ	$Range(\mathcal{C}), \{250, 500, 1000\}$	PH threshold
Rank-based bandits	$\mathcal{D}$	$\{.5\}$	Decay factor
All bandits	$\mathcal{C}$	$\{\{1, 5\}.10^{\{-4 \leq i \leq 2\}}\}$	Scaling factor

Table 1: AOS Hyper-parameters and value range

Table 2: Robustness analysis: best hyper-parameters configuration found for each technique, off-line tuned under different conditions.

Dim5	*F-AUC-B*	*PM-Ad*	*AP*	*DMAB*
separ.	C.5D.5W50	P.05α.9	P.2α.6β.6W500	C10γ.01W100
moder.	C.5D.5W50	P.05α.3	P.2α.9β.3W50	C.01G100W50
ill-cond.	C.5D.5W50	P0α.9	P.2α.6β.3W500	C100G1000W500
multi-m.	C.5D.5W50	P.05α.9	P.2α.9β.3W500	C100G1W50
weak-st.	C1D.5W500	P.05α.1	P.2α.9β.6W50	C100γ.1W50
all funct.	C.5D.5W50	P0α.9	P.2α.3β.6W500	C100γ.1W50

Dim20	*F-AUC-B*	*PM-Ad*	*AP*	*DMAB*
separ.	C.5D.5W50	P0α.9	P.2α.3β.1W500	C100γ.01W50
moder.	C.5D.5W50	P0α.9	P.2α.9β.3W500	C100γ.01W50
ill-cond.	C.5D.5W50	P0α.9	P.2α.6β.3W500	C100γ.01W50
multi-m.	C1D.5W50	P0α.3	P.2α.3β.1W100	C100γ.01W50
weak-st.	C.01D.5W50	P0α.9	P.2α.3β.3W100	C100γ.01W50
all funct.	C.5D.5W50	P0α.6	P.2α.1β.1W500	C100γ.1W50

configuration found for each technique on each of the analyzed cases is presented in Table 2.

This tuning experiment clearly demonstrates the robustness of *F-AUC-Bandit*, with $\{C = .5, D = .5, W = 50\}$ being always the best configuration, except for the *multi-modal* and *weak-structure* class functions, in which none of the techniques was able to perform well. The other rank-based techniques, AUC, F-SR and SR, are neglected here, as the conclusions (and the configuration found to be the best) are basically the same.

For the *PM-AdapSS-DE*, the advantages of using a relative instead of a raw reward are also shown on dimension 20, with always a very low value for p_{min} (represented by P), and a high one for the adaptation rate α.

For the *AP*, however, several configurations arrived to the end of the Racing process, all of them sharing $P.2$ and $W500$, but presenting all possible combinations for the adaptation rate α and the learning rate β.This could be seen as a good sign, possibly showing that this *AdapSS* combination was not sensitive w.r.t. its parameters; however, $P.2$ was found to be the best value for the minimal probability, what in fact means that the method presented a behavior really close to the uniform selection: as there are 4 strategies, the uniform would be equal to $P.25$, not mattering the other parameters. This is very possibly related to the credit assignment used, the extreme values [10], which uses the raw values of the fitness improvements, thus not scaling to the different fitness ranges provided by the tested functions.

The same kind of conclusions can be drawn for the tuning experiments of *DMAB*. The configurations found for the different situations were all quite similar, with $C100$, $\gamma \leq .1$ and $W50$. However, a very high scaling factor C was found to be the best, what means that much more weight was given to the exploration term of the UCB formula, *i.e.*, although knowing which is the current best strategy, the algorithm prefers to explore the others. Besides, a very low value for the Page-Hinkley change detection threshold γ means that the probability of having restarts during the search was really high, what also favors the exploration, thus dramatically decreasing the performance of the method.

To conclude with, just the fact that the same hyper-parameter tuning is found to be the best on different situations is not enough to state that a given technique is robust. Intuitively, if the final performance is as good as the uniform one, the configurations found are meaningless. The *F-AUC-Bandit* and the *PM-AdapSS-DE*, while presenting similar configurations for different situations, also perform very well, as shown in the empirical comparison presented in the following.

5.2 Final hyper-parameters setting

Besides checking their robustness, the same experiments were also used to define the hyper-parameter tuning to be used for each technique on the final experiments used by the empirical comparison.

For the *F-AUC-Bandit*, the following parameters were used: $\{C = .5, D = .5, W = 50\}$. For the *PM-AdapSS-DE*, although $\alpha.9$ was found to be the best value on dimension 5, $\alpha.6$ was not significantly different, being thus the configuration used in the final experiments on both dimensions (with $p_{min} = 0$). For the *AP*, the configuration used was the one that was found to be the best considering all functions, differently for each dimension. For *DMAB*, $\{C = 100, \gamma = .1, W = 50\}$ was used.

The same parameter values were used on all the experiments for each of the techniques, on each of the dimensions, thus the crafting effort (as defined in [18]) for all of them is equal to zero.

6 Results

Results from experiments according to [18] are presented in the following. The **expected running time (ERT)**, used in the figures and tables, depends on a given target function value, $f_t = f_{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [18, 23]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Section 6.1 presents a brief analysis, and figures and tables that summarize the empirical comparisons done. From Section 6.2 to 6.13, a pair-wise comparison is presented for each of the mentioned algorithms with relation to *F-AUC-Bandit*, with a table detailing the performance of each algorithm, and showing when there exists significant differences between them.

6.1 Results Summary

Although results for both dimensions 5 and 20 are presented in the following, finally it was found that the low number of evaluations necessary to achieve the target function value on dimension 5 was insufficient to allow the adaptive schemes to show a performance gain over the Uniform-DE. Thus, for this brief results analysis, just dimension 20 is considered.

The *F-AUC-Bandit* was firstly compared with the base technique, *i.e.*, the DE using a single mutation strategy. Four DE variants were tried, each of them implementing one of the strategies that compose the strategy set used by the adaptive schemes, referred to as DEn, summarized in Figure 2. The $DE4$ was not able of solving any of the functions at 20-D, thus being neglected in the following (although being possibly used by the adaptive scheme to attain the presented performance). All the other strategies achieved the target value on 60% of the trials for the *separable* functions, and on all trials for the *ill-conditioned* ones. For the *moderate* functions, both *F-AUC-Bandit* and $DE3$ were able to achieve 100% of success, while $DE1$ and $DE2$ got, respectively, 98% and 75%. The *F-AUC-Bandit* showed to be around 3 times faster than $DE1$ on the 3 analyzed function classes, while being around 20 times faster than $DE2$ on around 65%, 50% and 80% of the trials, respectively, for the *separable*, *moderate* and *ill-conditioned* function classes. $DE3$ was the best between the single strategies, performing 10 times faster than $DE2$, thus being around 2 times slower than *F-AUC-Bandit* at around the same rates.

The comparison with the 3 previously proposed *AdapSS* techniques, *PM-AdapSS-DE* [16], *AP* [28] and *DMAB* [7] (the two latter being fed by extreme rewards [10]), and with the uniform strategy selection, is summarized in Figure 3. *F-AUC-Bandit* showed to be around 1.5 times faster than the *Uniform-DE* in around 80% of the trials. The same speed factor was attained with relation to *AP*, at a rate of around 90% of the trials on the 3 function classes analyzed. It also showed to be around 3 times faster than *DMAB* on half of the trials, being at least around 1.5 times faster on all trials. The *PM-AdapSS-DE* was found to be the best one between the baseline techniques possibly because of the use of a relative instead of a raw reward. *F-AUC-Bandit* was around 1.5 times faster than *PM-AdapSS-DE* on just around 25% of the trials on the *separable*, and 40% for the *moderate* functions, with an even smaller performance difference being found for the *ill-conditioned* ones, although still being faster on around 75% of the functions.

By analyzing Figure 4, which present the aggregated performance of each algorithm over several functions, not much can be said about the comparison between *F-AUC-Bandit* and the other rank-based approaches (AUC, SR, F-SR), they present basically the same performance. However, when analyzing their performance independently for each function (what can be found in the pair-wise comparison tables found in the respective Sections in the following), it can be said that *F-AUC-Bandit* presents overall better results, although just a few significant differences can be found between them.

F-AUC-Bandit was not able to show competitive results with relation to the CMA-ES state-of-the-art optimizer, however it improved the performance of its base technique, the Differential Evolution. A better performance could be achieved by tuning more carefully the DE parameters, namely, the population size NP, the mutation scaling factor F, and the crossover rate CR; however, this is out of the scope of the current work.

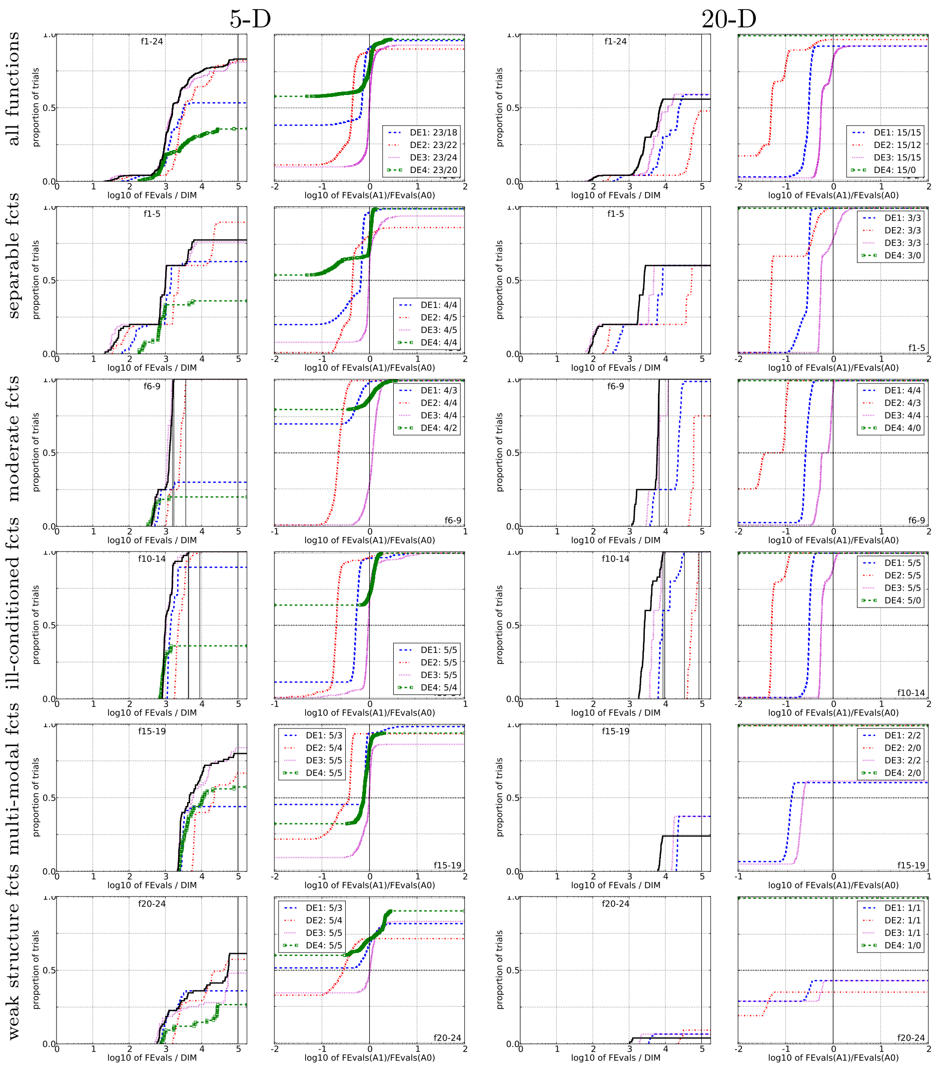

Figure 2: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right) of variants of DE versus *F-AUC-Bandit*. Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}}+10^{-8}$, with *F-AUC-Bandit* being represented by the black line. Right sub-columns: ECDF of FEval ratios of F-AUC divided by each of the techniques, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

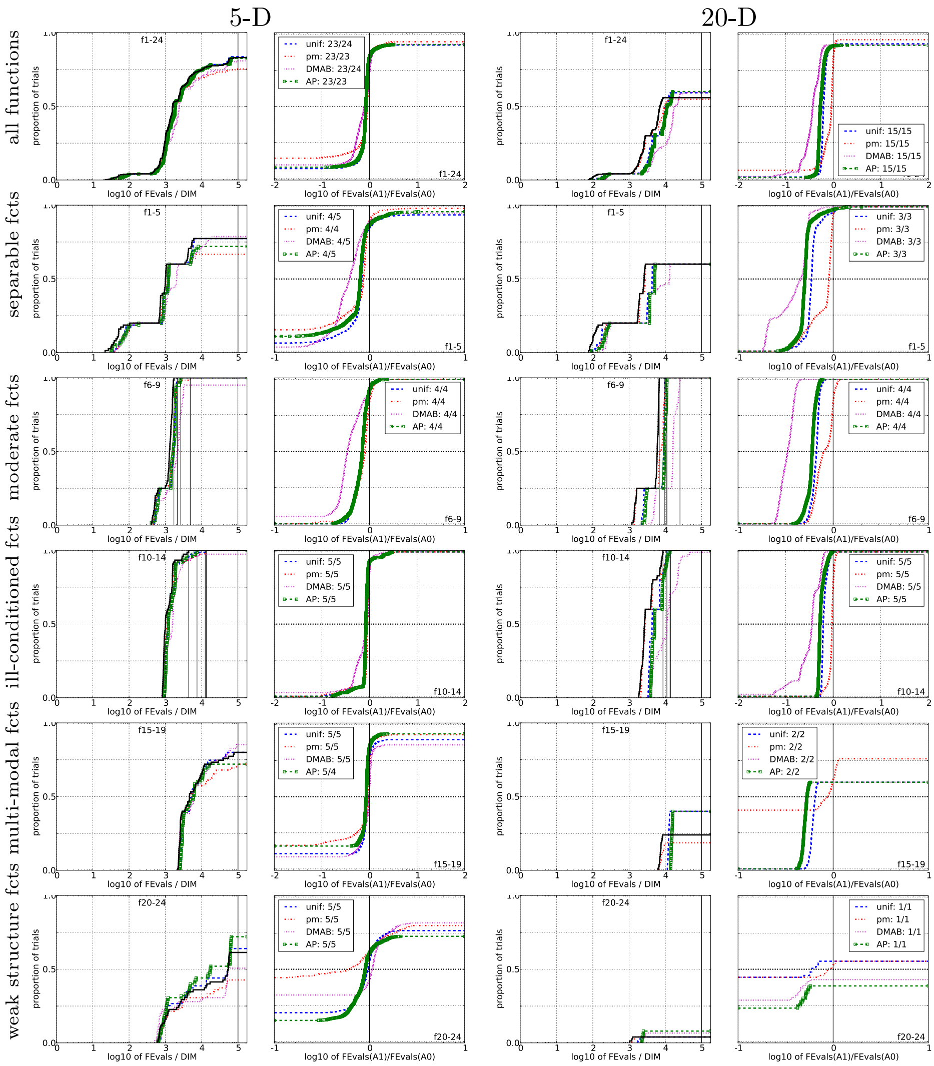

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right) of adaptive techniques versus *F-AUC-Bandit*. Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + 10^{-8}$, with *F-AUC-Bandit* being represented by the black line. Right sub-columns: ECDF of FEval ratios of F-AUC divided by each of the techniques, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

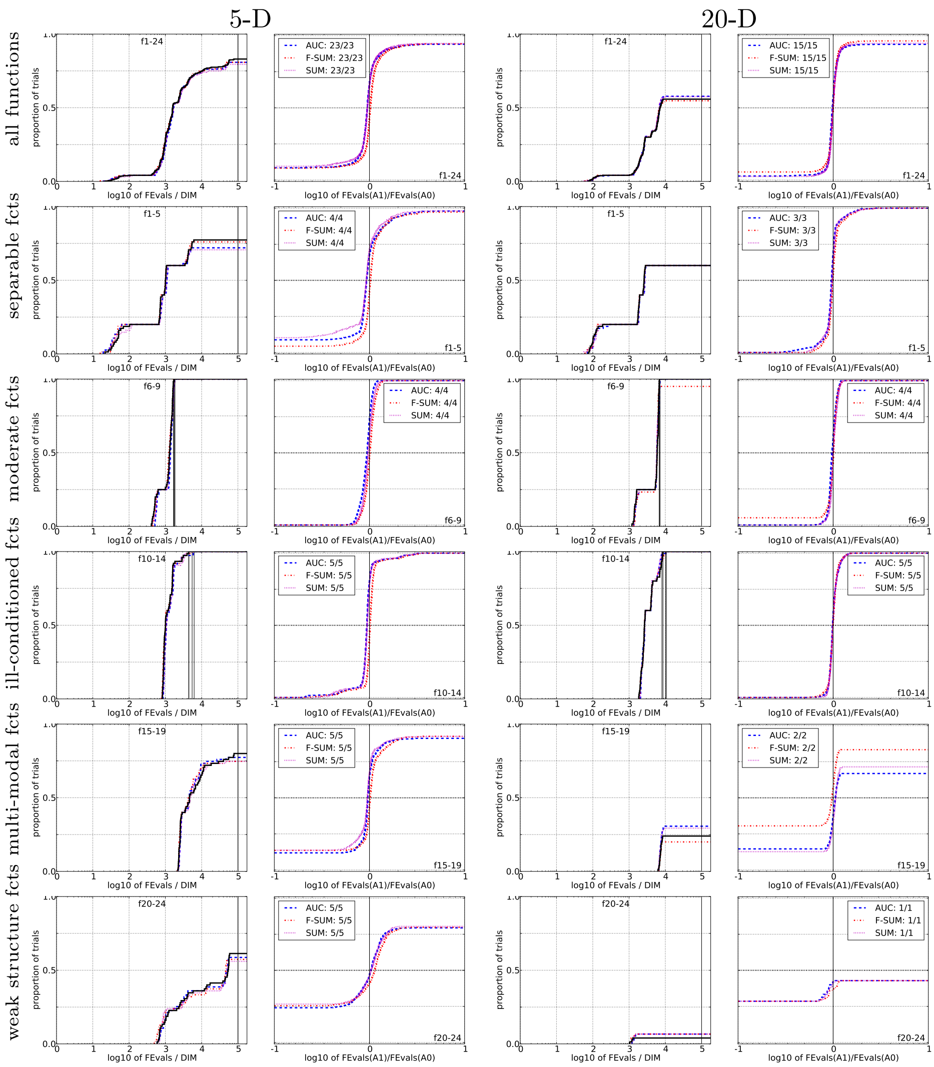

Figure 4: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right) of rank-based adaptive techniques versus *F-AUC-Bandit*. Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + 10^{-8}$, with *F-AUC-Bandit* being represented by the black line. Right sub-columns: ECDF of FEval ratios of F-AUC divided by each of the techniques, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

Separable functions

FEvals/D	10	1000	100000	p_s
F-AUC	43 (71)	**16652** (14217)	**51991** (4.27e5)	0.60
AUC	**0.84**(0.47)	1.1 (0.05)	1.0 (0.51)	0.60
F-SUM	1.1 (0.28)	**0.99**(0.07)	**0.96**(0.39)	0.60
SUM	**0.83**(0.41)	**1.0** (0.08)	**1.0** (0.40)	0.60
unif	1.0 (0.28)	1.6 (0.16)	1.6 (0.63)	0.60
DE1	1.0 (0.84)	3.3 (0.97)	3.2 (2.1)	0.60
DE2	1.00 (4.3)	20 (6.2)	21 (∞)	0.60
DE3	0.86 (0.14)	1.7 (0.56)	1.9 (2.6)	0.60
DE4	1.0 (0.39)	∞ (∞)	∞ (∞)	0.00
pm	1.0 (0.42)	1.1 (0.25)	1.1 (0.88)	0.60
DMAB	1.0 (0.42)	7.7 (6.8)	3.5 (∞)	0.60
AP	**0.82**(0.53)	1.9 (0.22)	1.9 (0.50)	0.60

Moderate functions

FEvals/D	10	1000	100000	p_s
F-AUC	110 (16)	**16589** (6570)	**1.20e5**(50995)	1.00
AUC	**0.66**(0.80)	1.1 (0.04)	**1.0** (0.08)	1.00
F-SUM	0.89 (0.58)	**0.99** (3.7)	2.2 (3.8)	0.95
SUM	**0.87**(0.60)	**1.1** (0.03)	**0.99**(0.07)	1.00
unif	**0.89**(0.35)	1.7 (0.23)	1.5 (0.12)	1.00
DE1	1.7 (1.1)	3.5 (0.58)	4.0 (1.1)	0.98
DE2	3.9 (3.8)	28 (3.4)	22 (∞)	0.75
DE3	0.92 (0.48)	1.9 (0.31)	1.5 (0.92)	1.00
DE4	1.1 (0.79)	1442 (1018)	∞ (.)	0.00
pm	1.1 (1.1)	1.1 (0.06)	1.3 (0.49)	1.00
DMAB	1.1 (5.6)	10 (7.0)	3.1 (0.76)	1.00
AP	1.1 (0.45)	1.9 (0.21)	1.6 (0.22)	1.00

Ill-Conditioned functions

FEvals/D	10	1000	100000	p_s
F-AUC	67 (125)	**19338** (314)	**52284** (52393)	1.00
AUC	**0.79**(0.29)	1.1 (0.05)	1.0 (0.08)	1.00
F-SUM	**0.69**(0.73)	**1.0** (0.02)	**1.0** (0.01)	1.00
SUM	0.88 (0.45)	**1.1** (0.01)	**0.99**(0.06)	1.00
unif	0.86 (0.54)	1.7 (0.08)	1.7 (0.24)	1.00
DE1	0.87 (0.55)	3.2 (0.20)	3.2 (0.23)	1.00
DE2	1.2 (2.7)	20 (1.9)	20 (3.4)	1.00
DE3	0.89 (0.61)	1.9 (0.15)	1.8 (0.35)	1.00
DE4	**0.69**(0.53)	∞ (.)	∞ (.)	0.00
pm	1.0 (0.44)	1.1 (0.03)	1.1 (0.11)	1.00
DMAB	1.3 (2.0)	6.1 (8.0)	3.9 (2.6)	0.99
AP	0.90 (0.64)	1.9 (0.14)	1.9 (0.32)	1.00

Table 3: Median ERT speed-up in 20-D (interquartile range in brackets) for a given budget of FEvals, for sets of functions f_{1-5}, f_{6-9} and f_{10-14}. The ERT speed-up is computed as the ratio of the ERT of the algorithms considered (row) over the ERT of F-AUC for the smallest function value attained by F-AUC after a budget of 10, 10^3, 10^5 times the dimension function evaluations or 10^{-8} if it was smaller. The best three values are in bold. The probability of success over all function instances for reaching the precision 10^{-8} is given in the rightmost column.

Multi-modal functions

FEvals/D	10	1000	100000	p_s
F-AUC	**28** (93)	**10555** (12664)	4.50e5 (1.51e6)	0.24
AUC	1.6 (1.1)	**1.0** (0.15)	**0.52** (0.58)	0.31
F-SUM	1.2 (0.35)	**1.0** (0.11)	1.1 (0.89)	0.20
SUM	1.1 (0.24)	1.1 (0.17)	**0.62** (0.31)	0.29
unif	1.2 (0.50)	1.7 (0.68)	1.2 (0.86)	0.40
DE1	**1.0** (0.98)	3.7 (2.0)	2.9 (4.2)	0.37
DE2	2.6 (4.8)	33 (10)	∞ (∞)	0.00
DE3	1.9 (1.2)	2.3 (1.1)	1.2 (1.6)	0.37
DE4	1.5 (0.74)	1.2 (0.67)	1.3 (∞)	0.00
pm	1.1 (0.42)	1.2 (0.28)	**0.44** (1.3)	0.19
DMAB	1.1 (1.0)	5.7 (2.3)	1.4 (3.6)	0.40
AP	**0.99** (1.2)	2.2 (0.52)	1.5 (1.3)	0.40

Weak-structure functions

FEvals/D	10	1000	100000	p_s
F-AUC	**8.4** (82)	**6138** (5359)	7.36e5 (9.08e5)	0.04
AUC	1.1 (0.18)	1.2 (5.4)	0.83 (0.31)	0.07
F-SUM	**1** (0.16)	**1.1** (6.7)	0.94 (0.48)	0.07
SUM	**1** (0.22)	1.2 (7.0)	**0.76** (0.34)	0.07
unif	1 (0.70)	4.6 (27)	0.95 (0.58)	0.04
DE1	1 (1.9)	3.6 (10)	**0.59** (12)	0.07
DE2	1 (3.8)	28 (28)	10 (∞)	0.09
DE3	1 (0.43)	2.5 (16)	1.0 (8.0)	0.07
DE4	1.1 (0.30)	**1.1** (6.5)	0.78 (1.0)	0.00
pm	1 (0.37)	1.2 (13)	**0.73** (0.91)	0.04
DMAB	1.7 (1.9)	10 (7.6)	4.9 (8.7)	0.07
AP	1 (0.29)	2.1 (0.50)	1.3 (1.0)	0.08

All functions

FEvals/D	10	1000	100000	p_s
F-AUC	66 (96)	**15273** (13036)	1.25e5 (5.41e5)	0.56
AUC	1.00 (0.65)	1.1 (0.08)	**0.97** (0.32)	0.58
F-SUM	1.1 (0.49)	**1.0** (0.09)	**1.00** (0.22)	0.55
SUM	**0.99** (0.37)	**1.1** (0.06)	**0.95** (0.36)	0.57
unif	**0.99** (0.44)	1.7 (0.27)	1.4 (0.58)	0.59
DE1	1.0 (0.98)	3.3 (0.64)	3.2 (2.7)	0.59
DE2	1.6 (4.3)	25 (11)	20 (∞)	0.48
DE3	**0.91** (0.50)	1.9 (0.50)	1.8 (1.3)	0.59
DE4	1.1 (0.53)	13 (∞)	∞ (∞)	0.00
pm	1.0 (0.39)	1.1 (0.10)	1.0 (0.85)	0.55
DMAB	1.2 (0.84)	7.7 (7.5)	3.4 (2.9)	0.59
AP	1 (0.57)	2.0 (0.27)	1.7 (0.59)	0.60

Table 4: Median ERT speed-up in 20-D (interquartile range in brackets) for a given budget of FEvals, for sets of functions f_{15-19}, f_{20-24}, f_{1-24}. The ERT speed-up is computed as the ratio of the ERT of the algorithms considered (row) over the ERT of F-AUC for the smallest function value attained by F-AUC after a budget of 10, 10^3, 10^5 times the dimension function evaluations or 10^{-8} if it was smaller. The best three values are in bold. The probability of success over all function instances for reaching the precision 10^{-8} is given in the rightmost column.

6.2 *F-AUC-Bandit* versus DE/rand/1

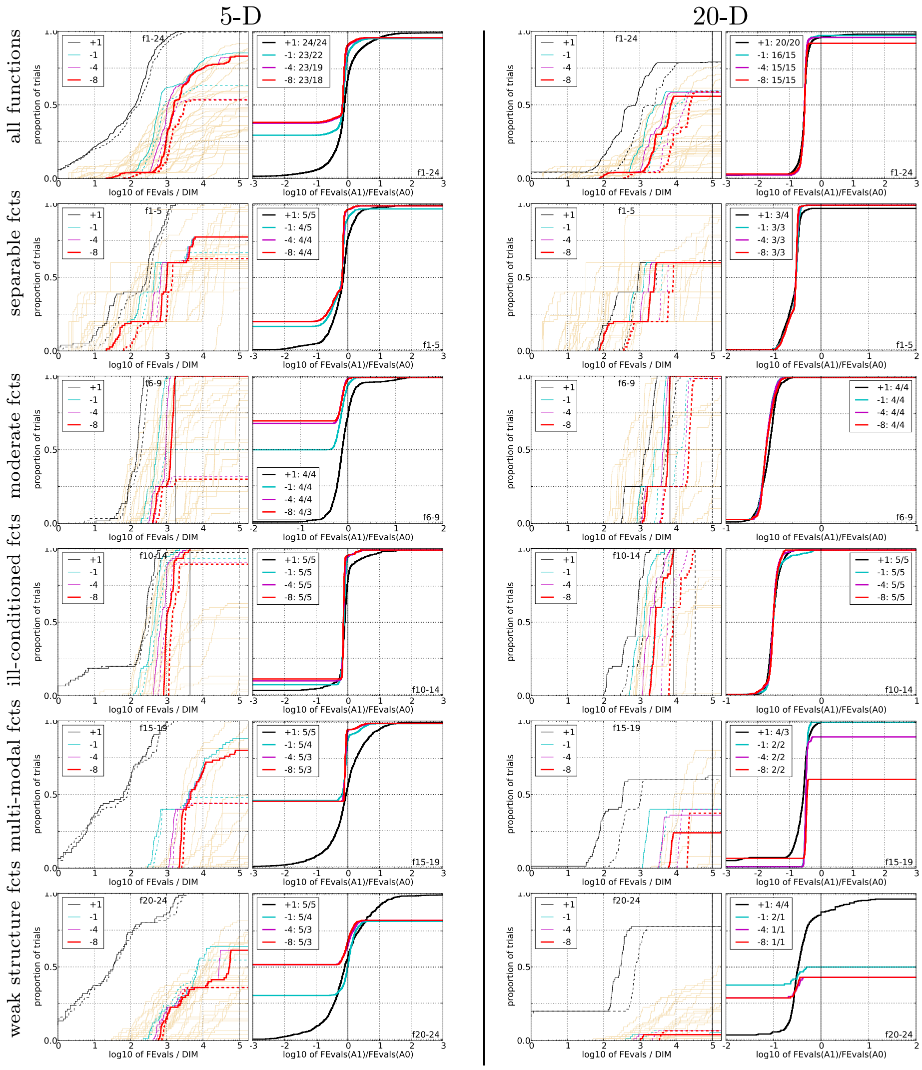

Figure 5: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and DE/rand/1 (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by DE/rand/1, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f_1	11	12	12	12	12	15/15
0:DE1	13	100	190	288	374	15/15
1:f-auc	6	**67**$^{\star 2}$	**132**$^{\star 3}$	**203**$^{\star 3}$	**266**$^{\star 3}$	15/15
f_2	83	88	90	92	94	15/15
0:DE1	23	36	48	59	70	15/15
1:f-auc	**18**$^{\star 3}$	**26**$^{\star 3}$	**35**$^{\star 3}$	**42**$^{\star 3}$	**50**$^{\star 3}$	15/15
f_3	716	1637	1646	1650	1654	15/15
0:DE1	4	849	1226	1980	1976	2/15
1:f-auc	3	59	60	**60***	**60***	13/15
f_4	809	1688	1817	1886	1903	15/15
0:DE1	6	4156	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f_5	10	10	10	10	10	15/15
0:DE1	35	72	73	73	73	15/15
1:f-auc	**15**$^{\star 2}$	**24**$^{\star 3}$	**24**$^{\star 3}$	**24**$^{\star 3}$	**24**$^{\star 3}$	15/15
f_6	114	281	580	1038	1332	15/15
0:DE1	9	1569	1.2e4	∞	∞ *5.0e5*	0/15
1:f-auc	6	**8**$^{\star 3}$	**7**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f_7	24	1171	1572	1572	1597	15/15
0:DE1	15	1	2	2	2	15/15
1:f-auc	13	**1***	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
f_8	73	336	391	410	422	15/15
0:DE1	16	9673	1.8e4	1.7e4	1.7e4	1/15
1:f-auc	13	**11**$^{\star 3}$	**11**$^{\star 3}$	**13**$^{\star 3}$	**14**$^{\star 3}$	15/15
f_9	35	214	300	335	369	15/15
0:DE1	34	4694	1.1e4	9712	8830	2/15
1:f-auc	24	**16**$^{\star 2}$	**16**$^{\star 3}$	**16**$^{\star 3}$	**17**$^{\star 3}$	15/15
f_{10}	349	574	626	829	880	15/15
0:DE1	5	5	7	6	7	15/15
1:f-auc	**4***	**4**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f_{11}	143	763	1177	1467	1673	15/15
0:DE1	7	3	3	3	3	15/15
1:f-auc	6	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f_{12}	108	371	461	1303	1494	15/15
0:DE1	742	687	961	444	388	7/15
1:f-auc	**22**$^{\star 2}$	13	14	6	7	15/15
f_{13}	132	250	1310	1752	2255	15/15
0:DE1	13	15	4	5	4	15/15
1:f-auc	**10**$^{\star 3}$	**11**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	15/15
f_{14}	10	58	139	251	476	15/15
0:DE1	2	21	18	15	11	15/15
1:f-auc	2	**15**$^{\star 3}$	**13**$^{\star 3}$	**11**$^{\star 3}$	**8**$^{\star 3}$	15/15
f_{15}	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:DE1	7	362	350	∞	∞ *5.0e5*	0/15
1:f-auc	5	**7**$^{\star 2}$	**7**$^{\star 2}$	**7**$^{\star 2}$	**7**$^{\star 2}$	12/15
f_{16}	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:DE1	9	382	193	173	167	3/15
1:f-auc	6	31	12	11	10	13/15
f_{17}	5	899	3669	6351	7934	15/15
0:DE1	2	3	2	2	2	15/15
1:f-auc	6	**2**$^{\star 2}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
f_{18}	103	3968	9280	1.1e4	1.2e4	15/15
0:DE1	5	0.9	0.8	1	1	15/15
1:f-auc	4	**0.8**$^{\star 2}$	**0.7**$^{\star 3}$	**0.9**$^{\star 3}$	**1.0**$^{\star 3}$	15/15
f_{19}	1	242	1.2e5	1.2e5	1.2e5	15/15
0:DE1	49	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	29	**1726**$^{\star 3}$	**11**$^{\star 3}$	**11**$^{\star 3}$	**10**$^{\star 3}$	5/15
f_{20}	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:DE1	11	5	5	5	5	10/15
1:f-auc	7	7	5	5	5	10/15
f_{21}	41	1674	1705	1729	1757	14/15
0:DE1	4	110	109	107	106	11/15
1:f-auc	5	110	109	107	106	11/15
f_{22}	71	938	1008	1040	1068	14/15
0:DE1	6	612	747	724	706	6/15
1:f-auc	5	803	747	724	706	6/15
f_{23}	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:DE1	2	11	225	∞	∞ *5.0e5*	0/15
1:f-auc	2	2	**3**$^{\star 3}$	**5**$^{\star 3}$	**7**$^{\star 3}$	15/15
f_{24}	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:DE1	6	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	4	**0.1**$^{\star 2}$	**0.2**$^{\star 2}$	**0.1**$^{\star 2}$	**0.1**$^{\star 2}$	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f_1	43	43	43	43	43	15/15
0:DE1	299	862	1419	1969	2543	15/15
1:f-auc	**93**$^{\star 3}$	**265**$^{\star 3}$	**431**$^{\star 3}$	**597**$^{\star 3}$	**763**$^{\star 3}$	15/15
f_2	385	387	390	391	393	15/15
0:DE1	149	211	272	333	395	15/15
1:f-auc	**48**$^{\star 3}$	**68**$^{\star 3}$	**86**$^{\star 3}$	**105**$^{\star 3}$	**123**$^{\star 3}$	15/15
f_3	5066	7635	7643	7646	7651	15/15
0:DE1	5709	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_4	4722	7666	7700	7758	1.4e5	9/15
0:DE1	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_5	41	41	41	41	41	15/15
0:DE1	230	266	267	267	267	15/15
1:f-auc	**42**$^{\star 3}$	**53**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	15/15
f_6	1296	3413	5220	6728	8409	15/15
0:DE1	73	56	56	58	59	15/15
1:f-auc	**19**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f_7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:DE1	17	6	5	5	5	15/15
1:f-auc	**5**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f_8	2039	4040	4219	4371	4484	15/15
0:DE1	80	121	125	126	129	14/15
1:f-auc	**23**$^{\star 3}$	**23**$^{\star 3}$	**24**$^{\star 3}$	**25**$^{\star 3}$	**26**$^{\star 3}$	15/15
f_9	1716	3277	3455	3594	3727	15/15
0:DE1	99	111	115	117	119	15/15
1:f-auc	**26**$^{\star 3}$	**27**$^{\star 3}$	**29**$^{\star 3}$	**30**$^{\star 3}$	**31**$^{\star 3}$	15/15
f_{10}	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:DE1	8	7	7	8	9	15/15
1:f-auc	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f_{11}	1002	6278	9762	1.2e4	1.5e4	15/15
0:DE1	21	7	7	8	8	15/15
1:f-auc	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f_{12}	1042	2740	4140	1.2e4	1.4e4	15/15
0:DE1	86	49	60	32	37	15/15
1:f-auc	**27**$^{\star 3}$	**19**$^{\star 3}$	**20**$^{\star 3}$	**9**$^{\star 3}$	**10**$^{\star 3}$	15/15
f_{13}	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:DE1	82	37	8	8	8	15/15
1:f-auc	**25**$^{\star 3}$	**11**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f_{14}	75	304	932	1648	1.6e4	15/15
0:DE1	110	125	73	59	8	15/15
1:f-auc	**33**$^{\star 3}$	**38**$^{\star 3}$	**23**$^{\star 3}$	**19**$^{\star 3}$	**3**$^{\star 3}$	15/15
f_{15}	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:DE1	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f_{16}	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:DE1	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_{17}	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:DE1	87	19	6	5	9	13/15
1:f-auc	**23**$^{\star 3}$	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**5***	13/15
f_{18}	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:DE1	35	5	3	3	3	15/15
1:f-auc	**11**$^{\star 3}$	**2**$^{\star 3}$	3	11	28	5/15
f_{19}	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:DE1	5830	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**1315**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f_{20}	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:DE1	144	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f_{21}	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:DE1	35	286	276	261	231	5/15
1:f-auc	**12**$^{\star 3}$	568	547	515	456	3/15
f_{22}	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:DE1	701	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**675***	∞	∞	∞	∞ *2.0e6*	0/15
f_{23}	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:DE1	3	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	**440**$^{\star 2}$	∞	∞	∞ *2.0e6*	0/15
f_{24}	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:DE1	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 5: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:DE1 is DE/rand/1 and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.3 *F-AUC-Bandit* versus DE/rand/2

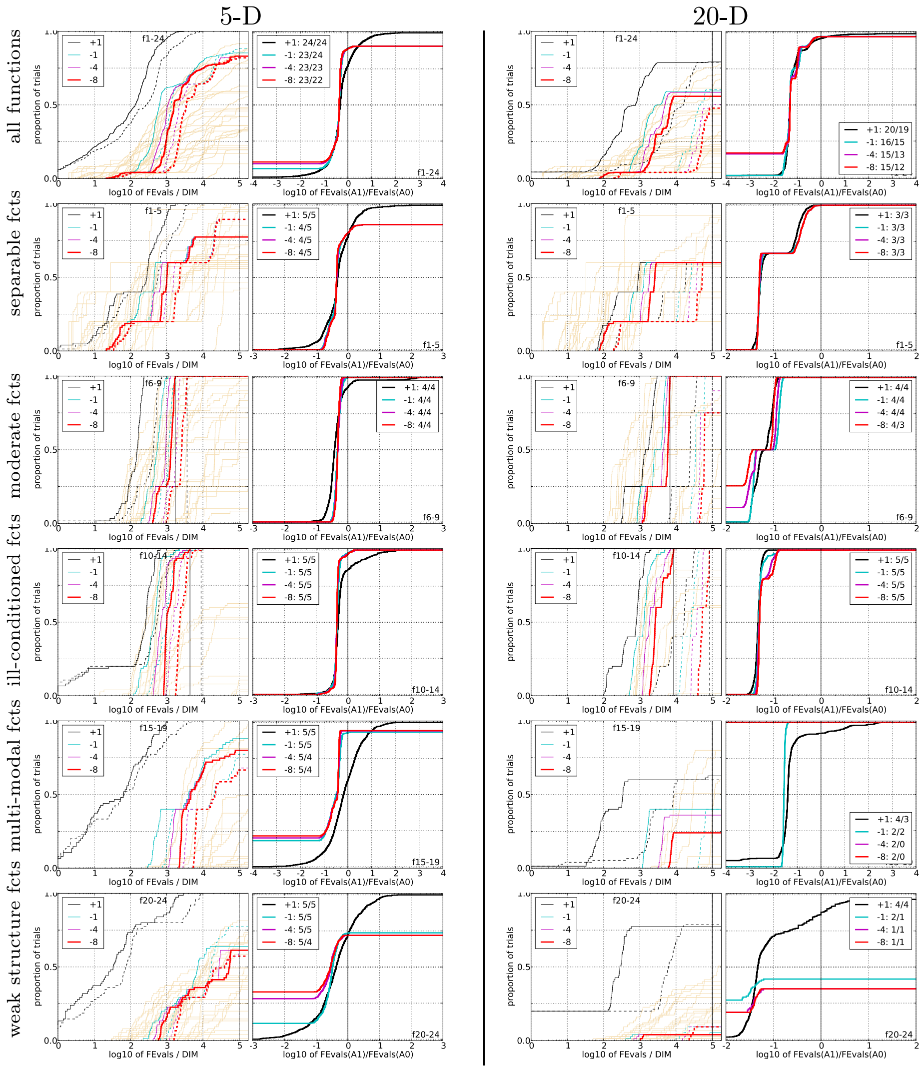

Figure 6: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and DE/rand/2 (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by DE/rand/2, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f$_1$	11	12	12	12	12	15/15
0:DE2	11	155	297	451	594	15/15
1:f-auc	6	**67**$^{\star 3}$	**132**$^{\star 3}$	**203**$^{\star 3}$	**266**$^{\star 3}$	15/15
f$_2$	83	88	90	92	94	15/15
0:DE2	38	58	78	96	113	15/15
1:f-auc	**18**$^{\star 3}$	**26**$^{\star 3}$	**35**$^{\star 3}$	**42**$^{\star 3}$	**50**$^{\star 3}$	15/15
f$_3$	716	1637	1646	1650	1654	15/15
0:DE2	13	56	57	58	59	15/15
1:f-auc	**3**$^{\star 2}$	59	60	60	60	13/15
f$_4$	809	1688	1817	1886	1903	15/15
0:DE2	11	**401**$^{\star 2}$	**374**$^{\star 2}$	**362**$^{\star 2}$	**360**$^{\star 2}$	7/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f$_5$	10	10	10	10	10	15/15
0:DE2	21	36	36	36	36	15/15
1:f-auc	15	24	24	24	24	15/15
f$_6$	114	281	580	1038	1332	15/15
0:DE2	20	19	15	12	12	15/15
1:f-auc	**6**$^{\star 3}$	**8**$^{\star 3}$	**7**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f$_7$	24	1171	1572	1572	1597	15/15
0:DE2	23	3	3	3	3	15/15
1:f-auc	13	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
f$_8$	73	336	391	410	422	15/15
0:DE2	33	20	22	25	29	15/15
1:f-auc	**13**$^{\star 2}$	**11**$^{\star 3}$	**11**$^{\star 3}$	**13**$^{\star 3}$	**14**$^{\star 3}$	15/15
f$_9$	35	214	300	335	369	15/15
0:DE2	63	31	27	30	32	15/15
1:f-auc	**24**$^{\star 3}$	**16**$^{\star 3}$	**16**$^{\star 3}$	**16**$^{\star 3}$	**17**$^{\star 3}$	15/15
f$_{10}$	349	574	626	829	880	15/15
0:DE2	9	9	11	11	12	15/15
1:f-auc	**4**$^{\star 3}$	**4**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f$_{11}$	143	763	1177	1467	1673	15/15
0:DE2	11	4	4	5	5	15/15
1:f-auc	**6**$^{\star 2}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f$_{12}$	108	371	461	1303	1494	15/15
0:DE2	58	29	30	13	13	15/15
1:f-auc	**22**$^{\star 3}$	**13***	**14***	**6***	**7***	15/15
f$_{13}$	132	250	1310	1752	2255	15/15
0:DE2	20	24	7	7	7	15/15
1:f-auc	**10**$^{\star 3}$	**11**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{14}$	10	58	139	251	476	15/15
0:DE2	1	34	30	26	18	15/15
1:f-auc	2	**15**$^{\star 3}$	**13**$^{\star 3}$	**11**$^{\star 3}$	**8**$^{\star 3}$	15/15
f$_{15}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:DE2	17	7	7	7	7	14/15
1:f-auc	5	7	7	7	7	12/15
f$_{16}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:DE2	12	226	92	83	94	6/15
1:f-auc	6	**31**$^{\star 2}$	**12**$^{\star 2}$	**11**$^{\star 2}$	**10**$^{\star 2}$	13/15
f$_{17}$	5	899	3669	6351	7934	15/15
0:DE2	6	5	3	3	3	15/15
1:f-auc	6	**2**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
f$_{18}$	103	3968	9280	1.1e4	1.2e4	15/15
0:DE2	7	2	2	2	2	15/15
1:f-auc	4	**0.8**$^{\star 3}$	**0.7**$^{\star 3}$	**0.9**$^{\star 3}$	**1.0**$^{\star 3}$	15/15
f$_{19}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:DE2	74	6956	∞	∞	∞ *5.0e5*	0/15
1:f-auc	29	1726	11	11	10	5/15
f$_{20}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:DE2	22	2	1	1	1	15/15
1:f-auc	7	7	5	5	5	10/15
f$_{21}$	41	1674	1705	1729	1757	14/15
0:DE2	7	26	27	27	27	14/15
1:f-auc	5	110	109	107	106	11/15
f$_{22}$	71	938	1008	1040	1068	14/15
0:DE2	8	474	442	429	419	8/15
1:f-auc	5	803	747	724	706	6/15
f$_{23}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:DE2	3	7	12	∞	∞ *5.0e5*	0/15
1:f-auc	2	**2**$^{\star 3}$	**3**$^{\star 3}$	**5**$^{\star 3}$	**7**$^{\star 3}$	15/15
f$_{24}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:DE2	16	0.2	0.1	0.1	0.1	6/15
1:f-auc	**4**$^{\star 3}$	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f$_1$	43	43	43	43	43	15/15
0:DE2	1820	5328	8748	1.2e4	1.6e4	15/15
1:f-auc	**93**$^{\star 3}$	**265**$^{\star 3}$	**431**$^{\star 3}$	**597**$^{\star 3}$	**763**$^{\star 3}$	15/15
f$_2$	385	387	390	391	393	15/15
0:DE2	961	1337	1718	2109	2487	15/15
1:f-auc	**48**$^{\star 3}$	**68**$^{\star 3}$	**86**$^{\star 3}$	**105**$^{\star 3}$	**123**$^{\star 3}$	15/15
f$_3$	5066	7635	7643	7646	7651	15/15
0:DE2	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_4$	4722	7666	7700	7758	1.4e5	9/15
0:DE2	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_5$	41	41	41	41	41	15/15
0:DE2	121	130	130	130	130	15/15
1:f-auc	**42**$^{\star 3}$	**53**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	15/15
f$_6$	1296	3413	5220	6728	8409	15/15
0:DE2	544	357	333	∞	∞ *2.0e6*	0/15
1:f-auc	**19**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f$_7$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:DE2	105	55	52	52	54	15/15
1:f-auc	**5**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f$_8$	2039	4040	4219	4371	4484	15/15
0:DE2	243	170	198	225	253	15/15
1:f-auc	**23**$^{\star 3}$	**23**$^{\star 3}$	**24**$^{\star 3}$	**25**$^{\star 3}$	**26**$^{\star 3}$	15/15
f$_9$	1716	3277	3455	3594	3727	15/15
0:DE2	280	207	240	272	303	15/15
1:f-auc	**26**$^{\star 3}$	**27**$^{\star 3}$	**29**$^{\star 3}$	**30**$^{\star 3}$	**31**$^{\star 3}$	15/15
f$_{10}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:DE2	50	49	45	48	56	15/15
1:f-auc	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{11}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:DE2	144	47	46	49	51	15/15
1:f-auc	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f$_{12}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:DE2	577	288	239	95	97	15/15
1:f-auc	**27**$^{\star 3}$	**19**$^{\star 3}$	**20**$^{\star 3}$	**9**$^{\star 3}$	**10**$^{\star 3}$	15/15
f$_{13}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:DE2	507	227	49	49	50	15/15
1:f-auc	**25**$^{\star 3}$	**11**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{14}$	75	304	932	1648	1.6e4	15/15
0:DE2	787	841	487	397	55	15/15
1:f-auc	**33**$^{\star 3}$	**38**$^{\star 3}$	**23**$^{\star 3}$	**19**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{15}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:DE2	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**474**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f$_{16}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:DE2	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_{17}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:DE2	352	227	∞	∞	∞ *2.0e6*	0/15
1:f-auc	23	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**5**$^{\star 3}$	13/15
f$_{18}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:DE2	281	64	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**11**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	**11**$^{\star 3}$	**28**$^{\star 3}$	5/15
f$_{19}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:DE2	3.7e4	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**1315**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f$_{20}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:DE2	937	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f$_{21}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:DE2	370	183	181	176	159	7/15
1:f-auc	**12**$^{\star 3}$	568	547	515	456	3/15
f$_{22}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:DE2	614	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f$_{23}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:DE2	2	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	**440**$^{\star 3}$	∞	∞	∞ *2.0e6*	0/15
f$_{24}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:DE2	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 6: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:DE2 is DE/rand/2 and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.4 *F-AUC-Bandit* versus DE/rand-to-best/2

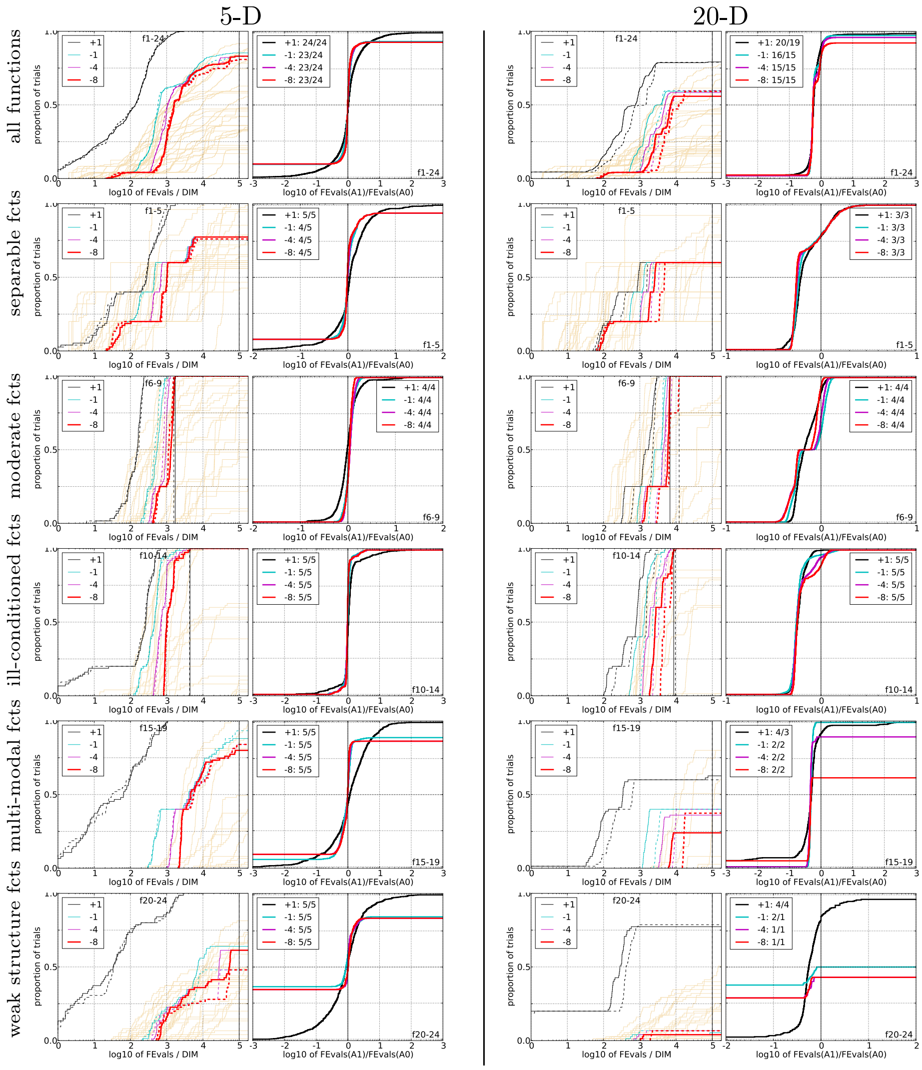

Figure 7: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and DE/rand-to-best/2 (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by DE/rand-to-best/2, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	15/15
0:DE3	6	69	133	202	272	15/15
1:f-auc	6	67	132	203	266	15/15
f2	83	88	90	92	94	15/15
0:DE3	17	26	34	43	51	15/15
1:f-auc	18	26	35	42	50	15/15
f3	716	1637	1646	1650	1654	15/15
0:DE3	4	166	166	166	166	10/15
1:f-auc	3	59	60	60	60	13/15
f4	809	1688	1817	1886	1903	15/15
0:DE3	4	1943	1805	1740	1725	2/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f5	10	10	10	10	10	15/15
0:DE3	11	18	18	18	18	15/15
1:f-auc	15	24	24	24	24	15/15
f6	114	281	580	1038	1332	15/15
0:DE3	6	8	**6***	5	**5***	15/15
1:f-auc	6	8	7	5	5	15/15
f7	24	1171	1572	1572	1597	15/15
0:DE3	12	1	1	1	1	15/15
1:f-auc	13	1	1	1	1	15/15
f8	73	336	391	410	422	15/15
0:DE3	13	9	10	11	13	15/15
1:f-auc	13	11	11	13	14	15/15
f9	35	214	300	335	369	15/15
0:DE3	28	14	13	14	15	15/15
1:f-auc	24	16	16	16	17	15/15
f10	349	574	626	829	880	15/15
0:DE3	4	4	5	5	5	15/15
1:f-auc	4	4	5	5	5	15/15
f11	143	763	1177	1467	1673	15/15
0:DE3	6	2	2	2	3	15/15
1:f-auc	6	2	2	2	2	15/15
f12	108	371	461	1303	1494	15/15
0:DE3	22	13	14	6	6	15/15
1:f-auc	22	13	14	6	7	15/15
f13	132	250	1310	1752	2255	15/15
0:DE3	10	11	3	3	3	15/15
1:f-auc	10	11	3	3	3	15/15
f14	10	58	139	251	476	15/15
0:DE3	2	15	14	12	8	15/15
1:f-auc	2	15	13	11	8	15/15
f15	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:DE3	5	8	7	7	7	12/15
1:f-auc	5	7	7	7	7	12/15
f16	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:DE3	7	26	10	9	9	14/15
1:f-auc	6	31	12	11	10	13/15
f17	5	899	3669	6351	7934	15/15
0:DE3	3	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
f18	103	3968	9280	1.1e4	1.2e4	15/15
0:DE3	3	0.7	0.6	0.8	1.0	15/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
f19	1	242	1.2e5	1.2e5	1.2e5	15/15
0:DE3	44	1209	7	7	7	7/15
1:f-auc	29	1726	11	11	10	5/15
f20	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:DE3	9	26	19	18	18	5/15
1:f-auc	7	7	5	5	5	10/15
f21	41	1674	1705	1729	1757	14/15
0:DE3	5	263	258	255	251	8/15
1:f-auc	5	110	109	107	106	11/15
f22	71	938	1008	1040	1068	14/15
0:DE3	4	802	746	724	705	6/15
1:f-auc	5	803	747	724	706	6/15
f23	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:DE3	2	2	3	5	6	15/15
1:f-auc	2	2	3	5	7	15/15
f24	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:DE3	4	0.5	0.3	0.3	0.3	2/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	15/15
0:DE3	166	489	818	1139	1465	15/15
1:f-auc	**93**$^{\star 3}$	**265**$^{\star 3}$	**431**$^{\star 3}$	**597**$^{\star 3}$	**763**$^{\star 3}$	15/15
f2	385	387	390	391	393	15/15
0:DE3	83	118	154	190	225	15/15
1:f-auc	**48**$^{\star 3}$	**68**$^{\star 3}$	**86**$^{\star 3}$	**105**$^{\star 3}$	**123**$^{\star 3}$	15/15
f3	5066	7635	7643	7646	7651	15/15
0:DE3	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f4	4722	7666	7700	7758	1.4e5	9/15
0:DE3	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f5	41	41	41	41	41	15/15
0:DE3	37	47	47	47	47	15/15
1:f-auc	42	53	54	54	54	15/15
f6	1296	3413	5220	6728	8409	15/15
0:DE3	33	25	24	25	25	15/15
1:f-auc	**19**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:DE3	9	4	3	3	4	15/15
1:f-auc	**5**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f8	2039	4040	4219	4371	4484	15/15
0:DE3	26	21	23	25	28	15/15
1:f-auc	23	23	24	25	26	15/15
f9	1716	3277	3455	3594	3727	15/15
0:DE3	30	26	28	31	33	15/15
1:f-auc	**26***	27	29	30	31	15/15
f10	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:DE3	4	4	4	4	5	15/15
1:f-auc	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f11	1002	6278	9762	1.2e4	1.5e4	15/15
0:DE3	11	4	4	4	5	15/15
1:f-auc	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f12	1042	2740	4140	1.2e4	1.4e4	15/15
0:DE3	49	27	24	10	10	15/15
1:f-auc	**27**$^{\star 3}$	19	20	9	10	15/15
f13	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:DE3	47	21	5	5	5	15/15
1:f-auc	**25**$^{\star 3}$	**11**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f14	75	304	932	1648	1.6e4	15/15
0:DE3	54	72	42	34	5	15/15
1:f-auc	**33**$^{\star 3}$	**38**$^{\star 3}$	**23**$^{\star 3}$	**19**$^{\star 3}$	**3**$^{\star 3}$	15/15
f15	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:DE3	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**474**$^{\star 2}$	∞	∞	∞	∞ *2.0e6*	0/15
f16	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:DE3	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f17	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:DE3	41	13	4	4	6	14/15
1:f-auc	23	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**5***	13/15
f18	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:DE3	20	4	2	2	3	14/15
1:f-auc	**11**$^{\star 3}$	**2**$^{\star 3}$	3	11	28	5/15
f19	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:DE3	3223	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**1315**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f20	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:DE3	85	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f21	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:DE3	19	285	275	259	230	5/15
1:f-auc	**12**$^{\star 3}$	568	547	515	456	3/15
f22	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:DE3	331	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f23	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:DE3	2	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	**440**$^{\star 2}$	∞	∞	∞ *2.0e6*	0/15
f24	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:DE3	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 7: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:DE3 is DE/rand-to-best/2 and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.5 *F-AUC-Bandit* versus DE/current-to-rand/1

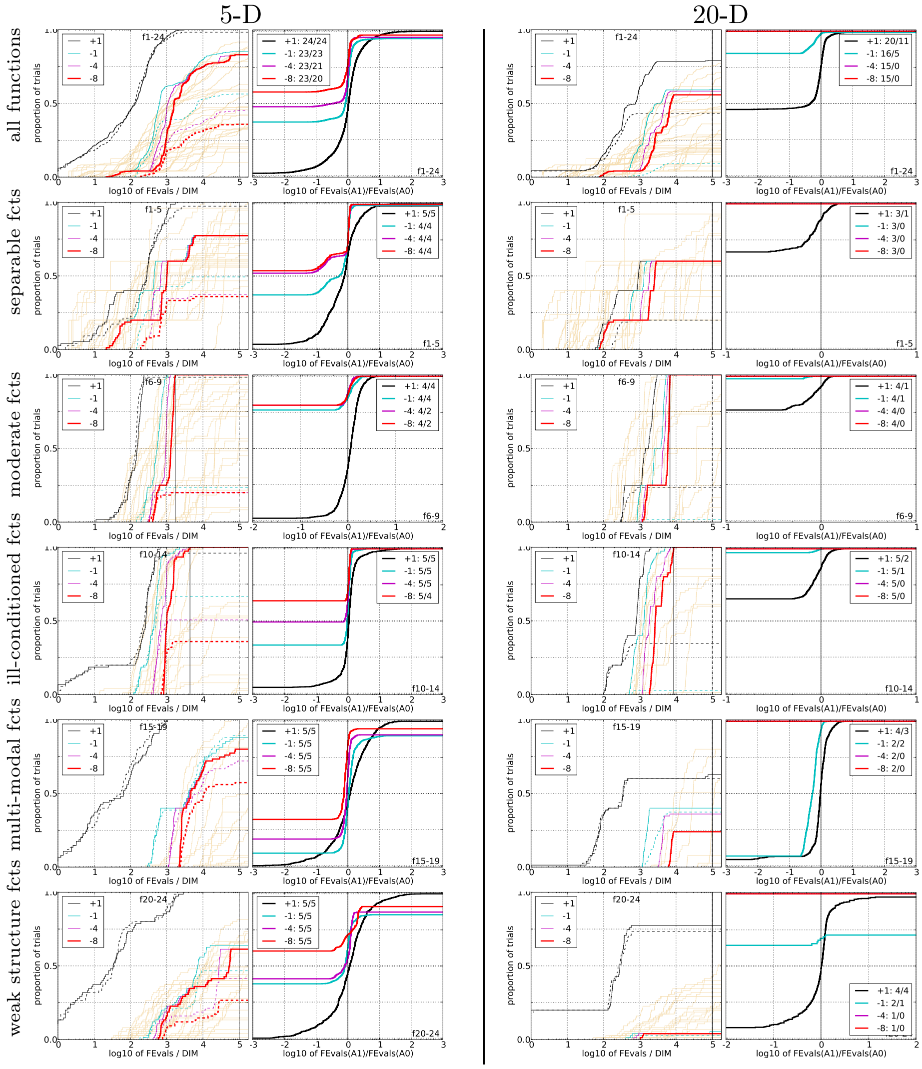

Figure 8: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and DE/current-to-rand/1 (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by DE/current-to-rand/1, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f$_1$	11	12	12	12	12	15/15
0:DE4	9	3001	1.0e4	1.5e4	1.5e4	11/15
1:f-auc	6	67	132	203	266	15/15
f$_2$	83	88	90	92	94	15/15
0:DE4	19	2083	3746	4799	6113	7/15
1:f-auc	18	26	35	42	50	15/15
f$_3$	716	1637	1646	1650	1654	15/15
0:DE4	3	623	1228	1983	1980	2/15
1:f-auc	3	59	60	**60***	**60**$^{\star 2}$	13/15
f$_4$	809	1688	1817	1886	1903	15/15
0:DE4	103	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f$_5$	10	10	10	10	10	15/15
0:DE4	91	5.7e4	5.7e4	5.7e4	5.7e4	7/15
1:f-auc	**15**$^{\star 3}$	**24**$^{\star 3}$	**24**$^{\star 3}$	**24**$^{\star 3}$	**24**$^{\star 3}$	15/15
f$_6$	114	281	580	1038	1332	15/15
0:DE4	319	2.5e4	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	**8**$^{\star 3}$	**7**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f$_7$	24	1171	1572	1572	1597	15/15
0:DE4	13	156	117	117	115	11/15
1:f-auc	13	1	1	1	1	15/15
f$_8$	73	336	391	410	422	15/15
0:DE4	11	2.1e4	∞	∞	∞ *5.0e5*	0/15
1:f-auc	13	**11**$^{\star 3}$	**11**$^{\star 3}$	**13**$^{\star 3}$	**14**$^{\star 3}$	15/15
f$_9$	35	214	300	335	369	15/15
0:DE4	20	3.3e4	2.3e4	2.1e4	1.9e4	1/15
1:f-auc	24	**16**$^{\star 2}$	**16**$^{\star 2}$	**16**$^{\star 2}$	**17**$^{\star 2}$	15/15
f$_{10}$	349	574	626	829	880	15/15
0:DE4	4	584	704	532	502	8/15
1:f-auc	4	4	5	5	5	15/15
f$_{11}$	143	763	1177	1467	1673	15/15
0:DE4	5	103	108	126	201	9/15
1:f-auc	6	2	2	2	2	15/15
f$_{12}$	108	371	461	1303	1494	15/15
0:DE4	353	3719	1.5e4	5379	∞ *5.0e5*	0/15
1:f-auc	22	13	**14**$^{\star 3}$	**6**$^{\star 3}$	**7**$^{\star 3}$	15/15
f$_{13}$	132	250	1310	1752	2255	15/15
0:DE4	591	1011	766	1145	890	3/15
1:f-auc	10	11	3	3	3	15/15
f$_{14}$	10	58	139	251	476	15/15
0:DE4	2	629	271	508	927	7/15
1:f-auc	2	15	13	11	8	15/15
f$_{15}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:DE4	4	14	18	17	28	7/15
1:f-auc	5	7	7	7	7	12/15
f$_{16}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:DE4	3	15	12	16	26	9/15
1:f-auc	6	31	12	11	10	13/15
f$_{17}$	5	899	3669	6351	7934	15/15
0:DE4	6	2	1	1	2	13/15
1:f-auc	6	2	1	1	**1***	15/15
f$_{18}$	103	3968	9280	1.1e4	1.2e4	15/15
0:DE4	4	10	9	8	7	13/15
1:f-auc	4	0.8	0.7	0.9	**1.0***	15/15
f$_{19}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:DE4	43	771	11	20	30	1/15
1:f-auc	29	1726	11	11	10	5/15
f$_{20}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:DE4	10	86	60	60	59	2/15
1:f-auc	7	7	5	5	5	10/15
f$_{21}$	41	1674	1705	1729	1757	14/15
0:DE4	3	151	259	256	328	7/15
1:f-auc	5	110	109	107	106	11/15
f$_{22}$	71	938	1008	1040	1068	14/15
0:DE4	3	802	1368	1927	3050	2/15
1:f-auc	5	803	747	724	706	6/15
f$_{23}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:DE4	2	2	**3**$^{\star 3}$	7	13	8/15
1:f-auc	2	2	3	5	7	15/15
f$_{24}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:DE4	3	0.5	0.3	0.3	0.6	1/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f$_1$	43	43	43	43	43	15/15
0:DE4	114	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	93	**265**$^{\star 3}$	**431**$^{\star 3}$	**597**$^{\star 3}$	**763**$^{\star 3}$	15/15
f$_2$	385	387	390	391	393	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**48**$^{\star 3}$	**68**$^{\star 3}$	**86**$^{\star 3}$	**105**$^{\star 3}$	**123**$^{\star 3}$	15/15
f$_3$	5066	7635	7643	7646	7651	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_4$	4722	7666	7700	7758	1.4e5	9/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_5$	41	41	41	41	41	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**42**$^{\star 3}$	**53**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	**54**$^{\star 3}$	15/15
f$_6$	1296	3413	5220	6728	8409	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**19**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f$_7$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:DE4	112	2949	∞	∞	∞ *2.0e6*	0/15
1:f-auc	5	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f$_8$	2039	4040	4219	4371	4484	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**23**$^{\star 3}$	**23**$^{\star 3}$	**24**$^{\star 3}$	**25**$^{\star 3}$	**26**$^{\star 3}$	15/15
f$_9$	1716	3277	3455	3594	3727	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**26**$^{\star 3}$	**27**$^{\star 3}$	**29**$^{\star 3}$	**30**$^{\star 3}$	**31**$^{\star 3}$	15/15
f$_{10}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{11}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:DE4	735	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	7	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f$_{12}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**27**$^{\star 3}$	**19**$^{\star 3}$	**20**$^{\star 3}$	**9**$^{\star 3}$	**10**$^{\star 3}$	15/15
f$_{13}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**25**$^{\star 3}$	**11**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{14}$	75	304	932	1648	1.6e4	15/15
0:DE4	34	4.3e4	∞	∞	∞ *2.0e6*	0/15
1:f-auc	33	**38**$^{\star 3}$	**23**$^{\star 3}$	**19**$^{\star 3}$	**3**$^{\star 3}$	15/15
f$_{15}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f$_{16}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f$_{17}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:DE4	22	13	426	∞	∞ *2.0e6*	0/15
1:f-auc	23	**7**$^{\star 2}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**5**$^{\star 3}$	13/15
f$_{18}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:DE4	11	19	∞	∞	∞ *2.0e6*	0/15
1:f-auc	11	**2**$^{\star 3}$	**3**$^{\star 3}$	**11**$^{\star 3}$	**28**$^{\star 3}$	5/15
f$_{19}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:DE4	1337	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
f$_{20}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:DE4	**36***	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	40	∞	∞	∞	∞ *2.0e6*	0/15
f$_{21}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:DE4	12	923	1913	∞	∞ *2.0e6*	0/15
1:f-auc	12	568	547	515	456	3/15
f$_{22}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:DE4	2157	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f$_{23}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:DE4	3	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
f$_{24}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:DE4	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 8: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:DE4 is DE/current-to-rand/1 and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.6 *F-AUC-Bandit* versus uniform DE

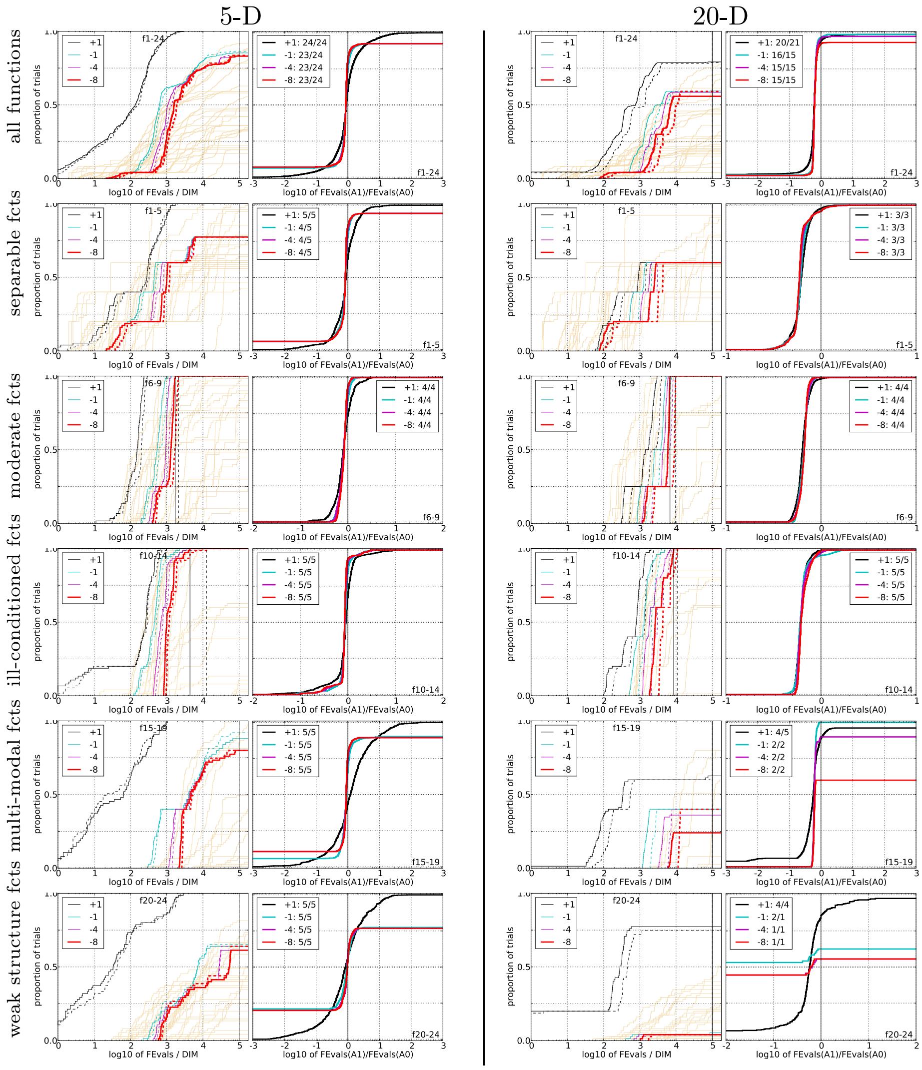

Figure 9: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and uniform DE (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by uniform DE, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	11	12	12	12	12	15/15
0:unif	9	80	155	236	313	15/15
1:f-auc	6	67	**132**^{*2}	**203**^{*2}	**266**^{*3}	15/15
$\mathbf{f_2}$	83	88	90	92	94	15/15
0:unif	20	30	40	50	58	15/15
1:f-auc	**18***	**26**^{*3}	**35**^{*3}	**42**^{*3}	**50**^{*3}	15/15
$\mathbf{f_3}$	716	1637	1646	1650	1654	15/15
0:unif	3	125	125	125	126	11/15
1:f-auc	3	59	60	60	60	13/15
$\mathbf{f_4}$	809	1688	1817	1886	1903	15/15
0:unif	5	1941	1803	1739	1723	2/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
$\mathbf{f_5}$	10	10	10	10	10	15/15
0:unif	20	35	35	35	35	15/15
1:f-auc	15	24	24	24	24	15/15
$\mathbf{f_6}$	114	281	580	1038	1332	15/15
0:unif	9	10	8	6	6	15/15
1:f-auc	**6***	**8**^{*2}	**7***	**5**^{*3}	**5**^{*3}	15/15
$\mathbf{f_7}$	24	1171	1572	1572	1597	15/15
0:unif	14	1	1	1	2	15/15
1:f-auc	13	1	1	1	**1**^{*2}	15/15
$\mathbf{f_8}$	73	336	391	410	422	15/15
0:unif	15	14	17	18	20	15/15
1:f-auc	13	11	**11**^{*2}	**13**^{*2}	**14**^{*2}	15/15
$\mathbf{f_9}$	35	214	300	335	369	15/15
0:unif	30	21	21	22	22	15/15
1:f-auc	24	**16***	**16**^{*2}	**16**^{*2}	**17**^{*2}	15/15
$\mathbf{f_{10}}$	349	574	626	829	880	15/15
0:unif	4	5	6	5	6	15/15
1:f-auc	4	**4***	**5***	**5**^{*2}	**5**^{*3}	15/15
$\mathbf{f_{11}}$	143	763	1177	1467	1673	15/15
0:unif	6	2	2	3	3	15/15
1:f-auc	6	**2**^{*2}	**2**^{*3}	**2**^{*3}	**2**^{*3}	15/15
$\mathbf{f_{12}}$	108	371	461	1303	1494	15/15
0:unif	28	21	22	10	11	15/15
1:f-auc	**22***	13	14	6	7	15/15
$\mathbf{f_{13}}$	132	250	1310	1752	2255	15/15
0:unif	11	13	4	4	4	15/15
1:f-auc	**10***	**11**^{*2}	**3**^{*3}	**3**^{*3}	**3**^{*3}	15/15
$\mathbf{f_{14}}$	10	58	139	251	476	15/15
0:unif	2	16	15	13	9	15/15
1:f-auc	2	15	**13***	**11**^{*3}	**8**^{*3}	15/15
$\mathbf{f_{15}}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:unif	5	3	3	3	3	14/15
1:f-auc	5	7	7	7	7	12/15
$\mathbf{f_{16}}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:unif	4	20	17	16	15	12/15
1:f-auc	6	31	12	11	10	13/15
$\mathbf{f_{17}}$	5	899	3669	6351	7934	15/15
0:unif	4	3	2	1	2	15/15
1:f-auc	6	**2**^{*2}	**1**^{*3}	**1**^{*3}	**1**^{*3}	15/15
$\mathbf{f_{18}}$	103	3968	9280	1.1e4	1.2e4	15/15
0:unif	4	0.8	0.7	0.9	1	15/15
1:f-auc	4	0.8	0.7	0.9	**1.0**^{*2}	15/15
$\mathbf{f_{19}}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:unif	35	1630	13	13	13	4/15
1:f-auc	29	1726	11	11	10	5/15
$\mathbf{f_{20}}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:unif	11	9	6	6	6	9/15
1:f-auc	7	7	5	5	5	10/15
$\mathbf{f_{21}}$	41	1674	1705	1729	1757	14/15
0:unif	4	76	75	74	74	12/15
1:f-auc	5	110	109	107	106	11/15
$\mathbf{f_{22}}$	71	938	1008	1040	1068	14/15
0:unif	7	469	437	424	414	8/15
1:f-auc	5	803	747	724	706	6/15
$\mathbf{f_{23}}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:unif	2	2	4	6	7	15/15
1:f-auc	2	2	3	5	7	15/15
$\mathbf{f_{24}}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:unif	4	0.2	0.1	0.1	0.1	4/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	43	43	43	43	43	15/15
0:unif	146	442	734	1027	1319	15/15
1:f-auc	**93**^{*3}	**265**^{*3}	**431**^{*3}	**597**^{*3}	**763**^{*3}	15/15
$\mathbf{f_2}$	385	387	390	391	393	15/15
0:unif	77	108	140	172	203	15/15
1:f-auc	**48**^{*3}	**68**^{*3}	**86**^{*3}	**105**^{*3}	**123**^{*3}	15/15
$\mathbf{f_3}$	5066	7635	7643	7646	7651	15/15
0:unif	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_4}$	4722	7666	7700	7758	1.4e5	9/15
0:unif	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_5}$	41	41	41	41	41	15/15
0:unif	69	85	86	86	86	15/15
1:f-auc	**42**^{*2}	**53***	**54***	**54***	**54***	15/15
$\mathbf{f_6}$	1296	3413	5220	6728	8409	15/15
0:unif	29	20	20	20	20	15/15
1:f-auc	**19**^{*3}	**14**^{*3}	**14**^{*3}	**14**^{*3}	**14**^{*3}	15/15
$\mathbf{f_7}$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:unif	8	3	3	3	3	15/15
1:f-auc	**5**^{*3}	**2**^{*3}	**2**^{*3}	**2**^{*3}	**2**^{*3}	15/15
$\mathbf{f_8}$	2039	4040	4219	4371	4484	15/15
0:unif	34	33	35	37	39	15/15
1:f-auc	**23**^{*3}	**23**^{*3}	**24**^{*3}	**25**^{*3}	**26**^{*3}	15/15
$\mathbf{f_9}$	1716	3277	3455	3594	3727	15/15
0:unif	41	41	43	45	46	15/15
1:f-auc	**26**^{*3}	**27**^{*3}	**29**^{*3}	**30**^{*3}	**31**^{*3}	15/15
$\mathbf{f_{10}}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:unif	4	4	4	4	5	15/15
1:f-auc	**2**^{*3}	**2**^{*3}	**2**^{*3}	**2**^{*3}	**3**^{*3}	15/15
$\mathbf{f_{11}}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:unif	11	4	4	4	4	15/15
1:f-auc	**7**^{*3}	**2**^{*3}	**2**^{*3}	**2**^{*3}	**2**^{*3}	15/15
$\mathbf{f_{12}}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:unif	44	25	26	12	13	15/15
1:f-auc	**27**^{*3}	19	**20***	**9**^{*2}	**10**^{*3}	15/15
$\mathbf{f_{13}}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:unif	43	19	4	4	4	15/15
1:f-auc	**25**^{*3}	**11**^{*3}	**2**^{*3}	**2**^{*3}	**3**^{*3}	15/15
$\mathbf{f_{14}}$	75	304	932	1648	1.6e4	15/15
0:unif	53	64	38	31	4	15/15
1:f-auc	**33**^{*3}	**38**^{*3}	**23**^{*3}	**19**^{*3}	**3**^{*3}	15/15
$\mathbf{f_{15}}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:unif	979	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{16}}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:unif	2.1e4	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{17}}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:unif	38	10	3	3	3	15/15
1:f-auc	23	**7**^{*3}	**2**^{*3}	**2**^{*3}	5	13/15
$\mathbf{f_{18}}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:unif	18	3	2	1	2	15/15
1:f-auc	**11**^{*3}	**2**^{*3}	3	11	28	5/15
$\mathbf{f_{19}}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:unif	2846	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**1315**^{*2}	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{20}}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:unif	76	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40**^{*3}	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{21}}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:unif	21	568	548	516	457	3/15
1:f-auc	**12**^{*3}	568	547	515	456	3/15
$\mathbf{f_{22}}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:unif	1580	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{23}}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:unif	3	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{24}}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:unif	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 9: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:unif is uniform DE and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.7 ***F-AUC-Bandit*** versus PM-AdapSS-DE

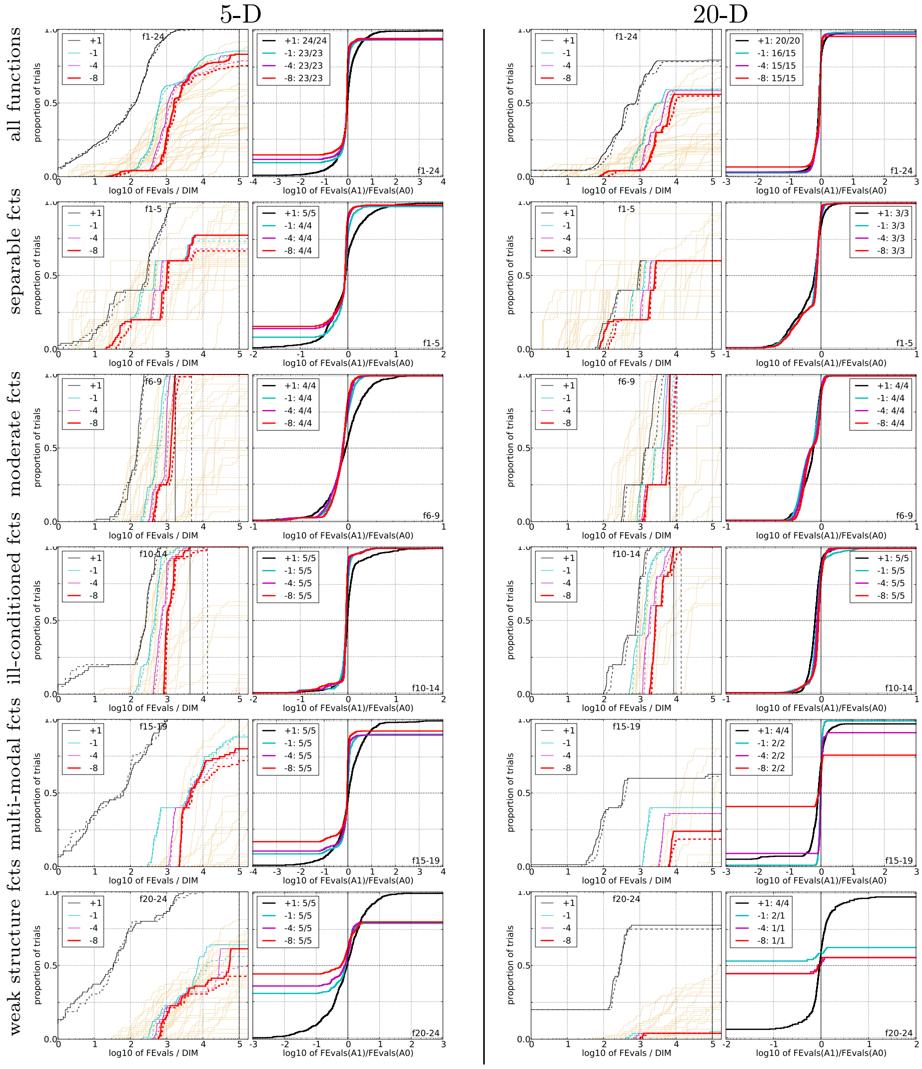

Figure 10: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and PM-AdapSS-DE (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by PM-AdapSS-DE, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	11	12	12	12	12	15/15
0:pmDE	5	74	145	223	292	15/15
1:f-auc	6	67	132	**203**^{*2}	**266**^{*2}	15/15
$\mathbf{f_2}$	83	88	90	92	94	15/15
0:pmDE	19	28	38	47	55	15/15
1:f-auc	18	**26**^{*2}	**35**^{*2}	**42**^{*2}	**50**^{*3}	15/15
$\mathbf{f_3}$	716	1637	1646	1650	1654	15/15
0:pmDE	5	166	362	621	621	5/15
1:f-auc	3	59	60	60	60	13/15
$\mathbf{f_4}$	809	1688	1817	1886	1903	15/15
0:pmDE	5	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
$\mathbf{f_5}$	10	10	10	10	10	15/15
0:pmDE	24	36	36	36	36	15/15
1:f-auc	15	24	24	24	24	15/15
$\mathbf{f_6}$	114	281	580	1038	1332	15/15
0:pmDE	8	9	7	6	6	15/15
1:f-auc	6	8	7	5	5	15/15
$\mathbf{f_7}$	24	1171	1572	1572	1597	15/15
0:pmDE	12	1	1	1	1	15/15
1:f-auc	13	1	1	1	1	15/15
$\mathbf{f_8}$	73	336	391	410	422	15/15
0:pmDE	13	15	19	20	21	15/15
1:f-auc	13	11	**11**^{*2}	**13**^{*2}	**14**^{*2}	15/15
$\mathbf{f_9}$	35	214	300	335	369	15/15
0:pmDE	25	21	21	21	22	15/15
1:f-auc	24	16	**16***	**16***	**17***	15/15
$\mathbf{f_{10}}$	349	574	626	829	880	15/15
0:pmDE	5	4	5	5	6	15/15
1:f-auc	4	4	5	**5***	**5**^{*2}	15/15
$\mathbf{f_{11}}$	143	763	1177	1467	1673	15/15
0:pmDE	6	2	2	2	3	15/15
1:f-auc	6	2	**2**^{*2}	**2***	**2**^{*3}	15/15
$\mathbf{f_{12}}$	108	371	461	1303	1494	15/15
0:pmDE	24	24	27	12	13	15/15
1:f-auc	22	13	14	6	7	15/15
$\mathbf{f_{13}}$	132	250	1310	1752	2255	15/15
0:pmDE	10	12	3	3	3	15/15
1:f-auc	10	11	**3**^{*2}	**3**^{*2}	**3**^{*2}	15/15
$\mathbf{f_{14}}$	10	58	139	251	476	15/15
0:pmDE	1	17	15	12	9	15/15
1:f-auc	2	15	**13***	**11***	**8***	15/15
$\mathbf{f_{15}}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:pmDE	5	3	3	8	7	12/15
1:f-auc	5	7	7	7	7	12/15
$\mathbf{f_{16}}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:pmDE	4	55	20	28	27	11/15
1:f-auc	6	31	12	11	10	13/15
$\mathbf{f_{17}}$	5	899	3669	6351	7934	15/15
0:pmDE	4	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
$\mathbf{f_{18}}$	103	3968	9280	1.1e4	1.2e4	15/15
0:pmDE	4	0.8	0.7	0.9	1	15/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
$\mathbf{f_{19}}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:pmDE	37	2054	19	19	29	1/15
1:f-auc	29	1726	11	11	10	5/15
$\mathbf{f_{20}}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:pmDE	11	9	7	7	7	8/15
1:f-auc	7	7	5	5	5	10/15
$\mathbf{f_{21}}$	41	1674	1705	1729	1757	14/15
0:pmDE	4	201	198	195	192	9/15
1:f-auc	5	110	109	107	106	11/15
$\mathbf{f_{22}}$	71	938	1008	1040	1068	14/15
0:pmDE	4	469	437	424	414	8/15
1:f-auc	5	803	747	724	706	6/15
$\mathbf{f_{23}}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:pmDE	2	12	14	21	27	5/15
1:f-auc	2	2	3	5	7	15/15
$\mathbf{f_{24}}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:pmDE	6	0.2	0.4	0.3	0.3	2/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	43	43	43	43	43	15/15
0:pmDE	101	291	465	646	827	15/15
1:f-auc	93	**265***	431	**597***	**763**^{*2}	15/15
$\mathbf{f_2}$	385	387	390	391	393	15/15
0:pmDE	52	73	93	112	132	15/15
1:f-auc	**48***	**68***	**86**^{*2}	**105**^{*2}	**123***	15/15
$\mathbf{f_3}$	5066	7635	7643	7646	7651	15/15
0:pmDE	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_4}$	4722	7666	7700	7758	1.4e5	9/15
0:pmDE	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_5}$	41	41	41	41	41	15/15
0:pmDE	82	96	96	96	96	15/15
1:f-auc	**42**^{*3}	**53**^{*3}	**54**^{*3}	**54**^{*3}	**54**^{*3}	15/15
$\mathbf{f_6}$	1296	3413	5220	6728	8409	15/15
0:pmDE	20	15	14	14	14	15/15
1:f-auc	19	14	14	14	14	15/15
$\mathbf{f_7}$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:pmDE	6	2	2	2	2	15/15
1:f-auc	**5**^{*2}	2	2	2	2	15/15
$\mathbf{f_8}$	2039	4040	4219	4371	4484	15/15
0:pmDE	35	38	39	40	41	15/15
1:f-auc	**23**^{*3}	**23**^{*3}	**24**^{*3}	**25**^{*3}	**26**^{*3}	15/15
$\mathbf{f_9}$	1716	3277	3455	3594	3727	15/15
0:pmDE	37	43	45	46	47	15/15
1:f-auc	**26**^{*3}	**27**^{*3}	**29**^{*3}	**30**^{*3}	**31**^{*3}	15/15
$\mathbf{f_{10}}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:pmDE	3	3	2	3	3	15/15
1:f-auc	**2***	**2**^{*2}	**2**^{*2}	**2**^{*3}	**3***	15/15
$\mathbf{f_{11}}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:pmDE	9	3	3	3	3	15/15
1:f-auc	**7***	2	**2***	**2***	**2***	15/15
$\mathbf{f_{12}}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:pmDE	29	20	24	12	13	15/15
1:f-auc	27	19	20	9	**10***	15/15
$\mathbf{f_{13}}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:pmDE	28	12	3	3	3	15/15
1:f-auc	**25**^{*2}	**11**^{*2}	**2***	2	**3***	15/15
$\mathbf{f_{14}}$	75	304	932	1648	1.6e4	15/15
0:pmDE	43	43	25	20	3	15/15
1:f-auc	**33**^{*2}	**38**^{*2}	23	19	3	15/15
$\mathbf{f_{15}}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:pmDE	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{16}}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:pmDE	2.2e4	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{17}}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:pmDE	23	7	2	2	18	7/15
1:f-auc	23	7	2	2	5	13/15
$\mathbf{f_{18}}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:pmDE	13	2	8	6	8	7/15
1:f-auc	11	2	3	11	28	5/15
$\mathbf{f_{19}}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:pmDE	1839	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{20}}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:pmDE	46	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	40	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{21}}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:pmDE	12	568	547	515	456	3/15
1:f-auc	12	568	547	515	456	3/15
$\mathbf{f_{22}}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:pmDE	1572	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{23}}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:pmDE	2	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{24}}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:pmDE	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 10: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:pmDE is PM-AdapSS-DE and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.8 *F-AUC-Bandit* versus AP

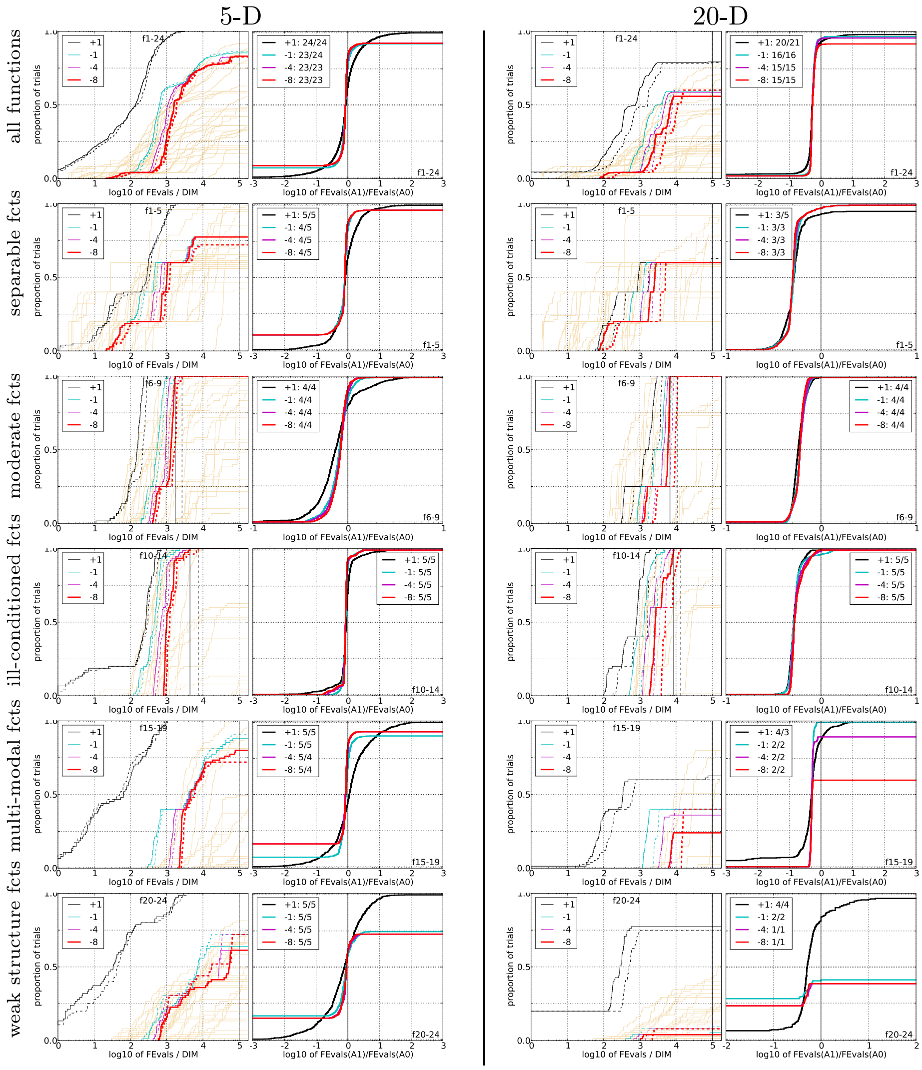

Figure 11: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and AP (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by AP, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	15/15
0:ap	6	85	171	242	324	15/15
1:f-auc	6	67	**132**$^{\star 3}$	**203**$^{\star 3}$	**266**$^{\star 3}$	15/15
f2	83	88	90	92	94	15/15
0:ap	22	31	41	50	59	15/15
1:f-auc	**18**$^{\star 2}$	**26**$^{\star 3}$	**35**$^{\star 3}$	**42**$^{\star 3}$	**50**$^{\star 3}$	15/15
f3	716	1637	1646	1650	1654	15/15
0:ap	4	282	281	281	281	8/15
1:f-auc	3	59	60	60	60	13/15
f4	809	1688	1817	1886	1903	15/15
0:ap	5	4168	3872	3732	3699	1/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f5	10	10	10	10	10	15/15
0:ap	22	37	37	37	37	15/15
1:f-auc	15	24	24	24	24	15/15
f6	114	281	580	1038	1332	15/15
0:ap	9	10	8	6	6	15/15
1:f-auc	6	**8**$^{\star 2}$	**7**$^{\star 2}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f7	24	1171	1572	1572	1597	15/15
0:ap	15	1	1	1	2	15/15
1:f-auc	13	1	1	1	**1***	15/15
f8	73	336	391	410	422	15/15
0:ap	16	14	17	18	20	15/15
1:f-auc	**13**$^{\star 2}$	11	**11**$^{\star 2}$	**13**$^{\star 2}$	**14**$^{\star 3}$	15/15
f9	35	214	300	335	369	15/15
0:ap	35	22	20	21	22	15/15
1:f-auc	**24**$^{\star 2}$	**16***	16	**16***	**17**$^{\star 2}$	15/15
f10	349	574	626	829	880	15/15
0:ap	5	5	6	5	6	15/15
1:f-auc	4	**4***	**5**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	15/15
f11	143	763	1177	1467	1673	15/15
0:ap	7	2	2	3	3	15/15
1:f-auc	6	**2**$^{\star 2}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f12	108	371	461	1303	1494	15/15
0:ap	27	13	16	8	9	15/15
1:f-auc	**22***	13	14	6	7	15/15
f13	132	250	1310	1752	2255	15/15
0:ap	11	13	4	4	4	15/15
1:f-auc	10	**11**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	**3**$^{\star 3}$	15/15
f14	10	58	139	251	476	15/15
0:ap	3	18	16	13	10	15/15
1:f-auc	2	15	**13**$^{\star 2}$	**11**$^{\star 3}$	**8**$^{\star 3}$	15/15
f15	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:ap	5	3	3	3	3	14/15
1:f-auc	5	7	7	7	7	12/15
f16	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:ap	3	19	29	26	25	10/15
1:f-auc	6	31	12	11	10	13/15
f17	5	899	3669	6351	7934	15/15
0:ap	3	3	2	1	2	15/15
1:f-auc	6	**2**$^{\star 2}$	**1**$^{\star 2}$	**1***	**1**$^{\star 3}$	15/15
f18	103	3968	9280	1.1e4	1.2e4	15/15
0:ap	5	0.8	0.8	1	1	15/15
1:f-auc	4	0.8	0.7	**0.9**$^{\star 3}$	**1.0**$^{\star 3}$	15/15
f19	1	242	1.2e5	1.2e5	1.2e5	15/15
0:ap	35	2153	∞	∞	∞ *5.0e5*	0/15
1:f-auc	29	1726	11	11	10	5/15
f20	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:ap	12	7	5	5	5	10/15
1:f-auc	7	7	5	5	5	10/15
f21	41	1674	1705	1729	1757	14/15
0:ap	6	48	47	47	46	13/15
1:f-auc	5	110	109	107	106	11/15
f22	71	938	1008	1040	1068	14/15
0:ap	5	269	251	244	238	10/15
1:f-auc	5	803	747	724	706	6/15
f23	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:ap	2	3	4	6	8	15/15
1:f-auc	2	2	**3**$^{\star 3}$	**5**$^{\star 2}$	**7**$^{\star 3}$	15/15
f24	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:ap	5	0.1	0.1	0.1	0.1	6/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	15/15
0:ap	178	520	858	1203	1543	15/15
1:f-auc	**93**$^{\star 3}$	**265**$^{\star 3}$	**431**$^{\star 3}$	**597**$^{\star 3}$	**763**$^{\star 3}$	15/15
f2	385	387	390	391	393	15/15
0:ap	90	128	166	203	240	15/15
1:f-auc	**48**$^{\star 3}$	**68**$^{\star 3}$	**86**$^{\star 3}$	**105**$^{\star 3}$	**123**$^{\star 3}$	15/15
f3	5066	7635	7643	7646	7651	15/15
0:ap	5901	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f4	4722	7666	7700	7758	1.4e5	9/15
0:ap	6315	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f5	41	41	41	41	41	15/15
0:ap	87	103	103	103	103	15/15
1:f-auc	**42**$^{\star 2}$	**53**$^{\star 2}$	**54**$^{\star 2}$	**54**$^{\star 2}$	**54**$^{\star 2}$	15/15
f6	1296	3413	5220	6728	8409	15/15
0:ap	33	24	23	23	23	15/15
1:f-auc	**19**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:ap	9	4	3	3	3	15/15
1:f-auc	**5**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f8	2039	4040	4219	4371	4484	15/15
0:ap	36	34	36	38	41	15/15
1:f-auc	**23**$^{\star 3}$	**23**$^{\star 3}$	**24**$^{\star 3}$	**25**$^{\star 3}$	**26**$^{\star 3}$	15/15
f9	1716	3277	3455	3594	3727	15/15
0:ap	44	43	45	47	49	15/15
1:f-auc	**26**$^{\star 3}$	**27**$^{\star 3}$	**29**$^{\star 3}$	**30**$^{\star 3}$	**31**$^{\star 3}$	15/15
f10	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:ap	5	5	4	5	5	15/15
1:f-auc	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f11	1002	6278	9762	1.2e4	1.5e4	15/15
0:ap	13	4	4	5	5	15/15
1:f-auc	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
f12	1042	2740	4140	1.2e4	1.4e4	15/15
0:ap	51	29	29	14	15	15/15
1:f-auc	**27**$^{\star 3}$	19	**20**$^{\star 2}$	**9**$^{\star 3}$	**10**$^{\star 3}$	15/15
f13	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:ap	51	23	5	5	5	15/15
1:f-auc	**25**$^{\star 3}$	**11**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**3**$^{\star 3}$	15/15
f14	75	304	932	1648	1.6e4	15/15
0:ap	65	74	44	36	5	15/15
1:f-auc	**33**$^{\star 3}$	**38**$^{\star 3}$	**23**$^{\star 3}$	**19**$^{\star 3}$	**3**$^{\star 3}$	15/15
f15	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:ap	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f16	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:ap	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f17	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:ap	43	12	4	4	3	15/15
1:f-auc	23	**7**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	5	13/15
f18	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:ap	21	3	2	2	2	15/15
1:f-auc	**11**$^{\star 3}$	**2**$^{\star 3}$	3	11	28	5/15
f19	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:ap	2743	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**1315***	∞	∞	∞	∞ *2.0e6*	0/15
f20	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:ap	85	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40**$^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
f21	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:ap	22	214	207	195	173	6/15
1:f-auc	**12**$^{\star 3}$	568	547	515	456	3/15
f22	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:ap	1586	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f23	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:ap	2	440	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
f24	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:ap	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 11: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:ap is AP and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.9 ***F-AUC-Bandit*** versus DMAB

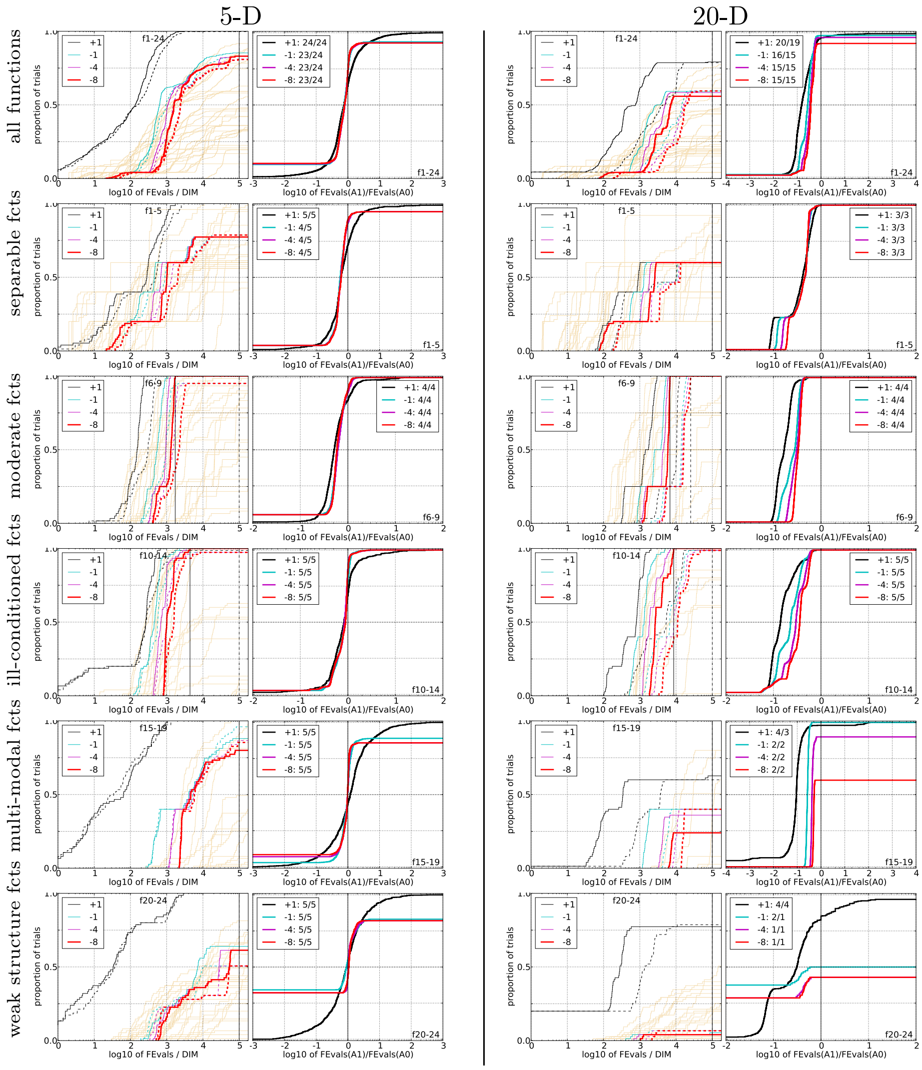

Figure 12: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and DMAB (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by DMAB, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_{1}}$	11	12	12	12	12	15/15
0:dmab	10	102	193	284	370	15/15
1:f-auc	6	$\mathbf{67}^{\star}$	$\mathbf{132}^{\star 2}$	$\mathbf{203}^{\star 2}$	$\mathbf{266}^{\star 2}$	15/15
$\mathbf{f_{2}}$	83	88	90	92	94	15/15
0:dmab	32	48	65	79	93	15/15
1:f-auc	$\mathbf{18}^{\star 3}$	$\mathbf{26}^{\star 3}$	$\mathbf{35}^{\star 3}$	$\mathbf{42}^{\star 3}$	$\mathbf{50}^{\star 3}$	15/15
$\mathbf{f_{3}}$	716	1637	1646	1650	1654	15/15
0:dmab	7	75	75	76	76	13/15
1:f-auc	3	$\mathbf{59}^{\star}$	$\mathbf{60}^{\star}$	$\mathbf{60}^{\star}$	$\mathbf{60}^{\star}$	13/15
$\mathbf{f_{4}}$	809	1688	1817	1886	1903	15/15
0:dmab	8	4173	3876	3736	3702	1/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
$\mathbf{f_{5}}$	10	10	10	10	10	15/15
0:dmab	21	37	37	37	37	15/15
1:f-auc	15	24	24	24	24	15/15
$\mathbf{f_{6}}$	114	281	580	1038	1332	15/15
0:dmab	14	289	144	83	67	13/15
1:f-auc	$\mathbf{6}^{\star}$	$\mathbf{8}^{\star 2}$	$\mathbf{7}^{\star}$	$\mathbf{5}^{\star}$	$\mathbf{5}^{\star}$	15/15
$\mathbf{f_{7}}$	24	1171	1572	1572	1597	15/15
0:dmab	19	2	2	2	2	15/15
1:f-auc	13	1	1	1	$\mathbf{1}^{\star}$	15/15
$\mathbf{f_{8}}$	73	336	391	410	422	15/15
0:dmab	25	124	110	109	109	14/15
1:f-auc	$\mathbf{13}^{\star 2}$	$\mathbf{11}^{\star 2}$	$\mathbf{11}^{\star 3}$	$\mathbf{13}^{\star 3}$	$\mathbf{14}^{\star 3}$	15/15
$\mathbf{f_{9}}$	35	214	300	335	369	15/15
0:dmab	47	25	23	25	26	15/15
1:f-auc	$\mathbf{24}^{\star}$	$\mathbf{16}^{\star 2}$	$\mathbf{16}^{\star}$	$\mathbf{16}^{\star 2}$	$\mathbf{17}^{\star 2}$	15/15
$\mathbf{f_{10}}$	349	574	626	829	880	15/15
0:dmab	7	6	8	8	9	15/15
1:f-auc	$\mathbf{4}^{\star 2}$	$\mathbf{4}^{\star 3}$	$\mathbf{5}^{\star 3}$	$\mathbf{5}^{\star 3}$	$\mathbf{5}^{\star 3}$	15/15
$\mathbf{f_{11}}$	143	763	1177	1467	1673	15/15
0:dmab	7	3	3	3	3	15/15
1:f-auc	6	2	$\mathbf{2}^{\star 2}$	2	$\mathbf{2}^{\star}$	15/15
$\mathbf{f_{12}}$	108	371	461	1303	1494	15/15
0:dmab	372	227	189	69	62	13/15
1:f-auc	$\mathbf{22}^{\star 2}$	13	14	6	7	15/15
$\mathbf{f_{13}}$	132	250	1310	1752	2255	15/15
0:dmab	10	11	3	3	3	15/15
1:f-auc	10	11	3	3	3	15/15
$\mathbf{f_{14}}$	10	58	139	251	476	15/15
0:dmab	2	17	14	12	9	15/15
1:f-auc	2	15	13	11	8	15/15
$\mathbf{f_{15}}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:dmab	6	3	3	3	3	14/15
1:f-auc	5	7	7	7	7	12/15
$\mathbf{f_{16}}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:dmab	5	24	14	12	12	13/15
1:f-auc	6	31	12	11	10	13/15
$\mathbf{f_{17}}$	5	899	3669	6351	7934	15/15
0:dmab	3	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
$\mathbf{f_{18}}$	103	3968	9280	1.1e4	1.2e4	15/15
0:dmab	3	0.7	0.6	0.8	4	14/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
$\mathbf{f_{19}}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:dmab	50	1129	6	6	6	8/15
1:f-auc	29	1726	11	11	10	5/15
$\mathbf{f_{20}}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:dmab	8	36	25	25	25	4/15
1:f-auc	7	7	5	5	5	10/15
$\mathbf{f_{21}}$	41	1674	1705	1729	1757	14/15
0:dmab	5	200	197	194	191	9/15
1:f-auc	5	110	109	107	106	11/15
$\mathbf{f_{22}}$	71	938	1008	1040	1068	14/15
0:dmab	5	469	436	423	413	8/15
1:f-auc	5	803	747	724	706	6/15
$\mathbf{f_{23}}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:dmab	2	3	3	5	7	15/15
1:f-auc	2	2	3	5	7	15/15
$\mathbf{f_{24}}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:dmab	5	0.5	0.3	0.3	0.3	2/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_{1}}$	43	43	43	43	43	15/15
0:dmab	155	494	816	1145	1471	15/15
1:f-auc	$\mathbf{93}^{\star 3}$	$\mathbf{265}^{\star 3}$	$\mathbf{431}^{\star 3}$	$\mathbf{597}^{\star 3}$	$\mathbf{763}^{\star 3}$	15/15
$\mathbf{f_{2}}$	385	387	390	391	393	15/15
0:dmab	390	430	465	501	536	15/15
1:f-auc	$\mathbf{48}^{\star 3}$	$\mathbf{68}^{\star 3}$	$\mathbf{86}^{\star 3}$	$\mathbf{105}^{\star 3}$	$\mathbf{123}^{\star 3}$	15/15
$\mathbf{f_{3}}$	5066	7635	7643	7646	7651	15/15
0:dmab	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{4}}$	4722	7666	7700	7758	1.4e5	9/15
0:dmab	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{5}}$	41	41	41	41	41	15/15
0:dmab	113	124	124	124	124	15/15
1:f-auc	$\mathbf{42}^{\star 3}$	$\mathbf{53}^{\star 3}$	$\mathbf{54}^{\star 3}$	$\mathbf{54}^{\star 3}$	$\mathbf{54}^{\star 3}$	15/15
$\mathbf{f_{6}}$	1296	3413	5220	6728	8409	15/15
0:dmab	154	98	71	60	52	15/15
1:f-auc	$\mathbf{19}^{\star 3}$	$\mathbf{14}^{\star 3}$	$\mathbf{14}^{\star 3}$	$\mathbf{14}^{\star 3}$	$\mathbf{14}^{\star 3}$	15/15
$\mathbf{f_{7}}$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:dmab	41	8	6	6	6	15/15
1:f-auc	$\mathbf{5}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	15/15
$\mathbf{f_{8}}$	2039	4040	4219	4371	4484	15/15
0:dmab	108	70	71	71	72	15/15
1:f-auc	$\mathbf{23}^{\star 3}$	$\mathbf{23}^{\star 3}$	$\mathbf{24}^{\star 3}$	$\mathbf{25}^{\star 3}$	$\mathbf{26}^{\star 3}$	15/15
$\mathbf{f_{9}}$	1716	3277	3455	3594	3727	15/15
0:dmab	122	83	83	83	84	15/15
1:f-auc	$\mathbf{26}^{\star 3}$	$\mathbf{27}^{\star 3}$	$\mathbf{29}^{\star 3}$	$\mathbf{30}^{\star 3}$	$\mathbf{31}^{\star 3}$	15/15
$\mathbf{f_{10}}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:dmab	24	20	17	17	19	15/15
1:f-auc	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{3}^{\star 3}$	15/15
$\mathbf{f_{11}}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:dmab	32	10	10	11	11	15/15
1:f-auc	$\mathbf{7}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	15/15
$\mathbf{f_{12}}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:dmab	415	169	121	44	41	14/15
1:f-auc	$\mathbf{27}^{\star 3}$	$\mathbf{19}^{\star 3}$	$\mathbf{20}^{\star 3}$	$\mathbf{9}^{\star 3}$	$\mathbf{10}^{\star 3}$	15/15
$\mathbf{f_{13}}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:dmab	172	50	9	8	7	15/15
1:f-auc	$\mathbf{25}^{\star 3}$	$\mathbf{11}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{3}^{\star 3}$	15/15
$\mathbf{f_{14}}$	75	304	932	1648	1.6e4	15/15
0:dmab	187	107	51	39	5	15/15
1:f-auc	$\mathbf{33}^{\star 3}$	$\mathbf{38}^{\star 3}$	$\mathbf{23}^{\star 3}$	$\mathbf{19}^{\star 3}$	$\mathbf{3}^{\star 3}$	15/15
$\mathbf{f_{15}}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:dmab	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	$\mathbf{474}^{\star}$	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{16}}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:dmab	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{17}}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:dmab	254	24	5	4	3	15/15
1:f-auc	$\mathbf{23}^{\star 2}$	$\mathbf{7}^{\star 3}$	$\mathbf{2}^{\star 3}$	$\mathbf{2}^{\star 3}$	5	13/15
$\mathbf{f_{18}}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:dmab	124	7	3	2	2	15/15
1:f-auc	$\mathbf{11}^{\star 3}$	$\mathbf{2}^{\star 3}$	3	11	28	5/15
$\mathbf{f_{19}}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:dmab	1.4e4	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	$\mathbf{1315}^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{20}}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:dmab	583	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	$\mathbf{40}^{\star 3}$	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{21}}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:dmab	59	286	276	260	230	5/15
1:f-auc	$\mathbf{12}^{\star 3}$	568	547	515	456	3/15
$\mathbf{f_{22}}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:dmab	408	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{23}}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:dmab	2	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	$\mathbf{440}^{\star 2}$	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{24}}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:dmab	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 12: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:dmab is DMAB and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.10 *F-AUC-Bandit* versus AUC-Bandit

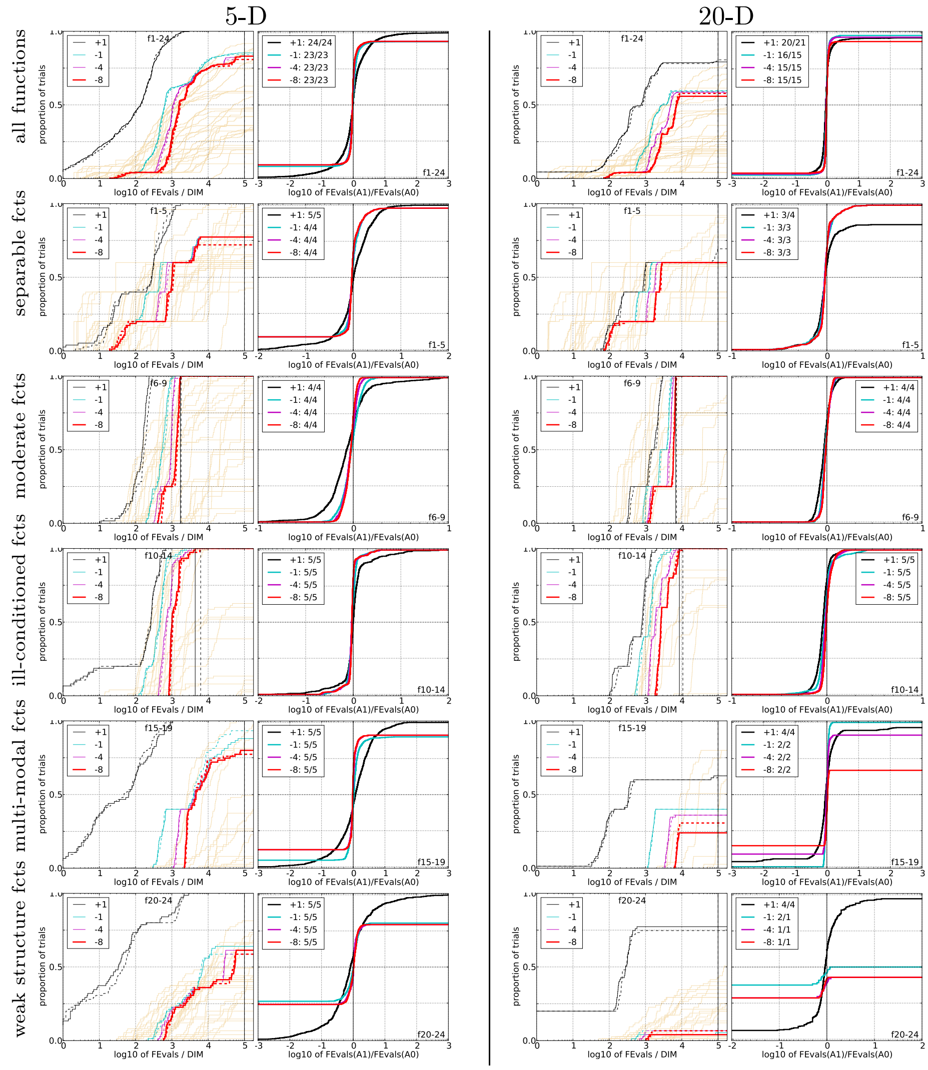

Figure 13: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and AUC-Bandit (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by AUC-Bandit, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f_1	11	12	12	12	12	15/15
0:auc	8	71	144	209	281	15/15
1:f-auc	6	67	132	203	266	15/15
f_2	83	88	90	92	94	15/15
0:auc	20	28	38	46	54	15/15
1:f-auc	18	**26***	**35***	**42***	**50***	15/15
f_3	716	1637	1646	1650	1654	15/15
0:auc	3	215	214	214	214	9/15
1:f-auc	3	59	60	60	60	13/15
f_4	809	1688	1817	1886	1903	15/15
0:auc	4	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f_5	10	10	10	10	10	15/15
0:auc	12	20	20	20	20	15/15
1:f-auc	15	24	24	24	24	15/15
f_6	114	281	580	1038	1332	15/15
0:auc	8	8	7	5	6	15/15
1:f-auc	6	8	7	5	5	15/15
f_7	24	1171	1572	1572	1597	15/15
0:auc	14	1	1	1	2	15/15
1:f-auc	13	1	**1***	**1***	1	15/15
f_8	73	336	391	410	422	15/15
0:auc	15	11	12	14	15	15/15
1:f-auc	13	11	11	13	14	15/15
f_9	35	214	300	335	369	15/15
0:auc	31	19	17	17	18	15/15
1:f-auc	24	16	16	16	17	15/15
f_{10}	349	574	626	829	880	15/15
0:auc	5	4	5	5	6	15/15
1:f-auc	4	4	5	5	5	15/15
f_{11}	143	763	1177	1467	1673	15/15
0:auc	6	2	2	2	3	15/15
1:f-auc	6	2	**2***	2	**2***	15/15
f_{12}	108	371	461	1303	1494	15/15
0:auc	24	15	16	7	8	15/15
1:f-auc	22	13	14	6	7	15/15
f_{13}	132	250	1310	1752	2255	15/15
0:auc	10	12	3	3	3	15/15
1:f-auc	10	11	**3***	**3**$^{\star 2}$	**3***	15/15
f_{14}	10	58	139	251	476	15/15
0:auc	2	15	15	12	9	15/15
1:f-auc	2	15	13	11	8	15/15
f_{15}	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:auc	4	1	1	1	1	15/15
1:f-auc	5	7	7	7	7	12/15
f_{16}	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:auc	4	15	21	19	19	11/15
1:f-auc	6	31	12	11	10	13/15
f_{17}	5	899	3669	6351	7934	15/15
0:auc	4	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
f_{18}	103	3968	9280	1.1e4	1.2e4	15/15
0:auc	4	0.7	0.7	0.9	1	15/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
f_{19}	1	242	1.2e5	1.2e5	1.2e5	15/15
0:auc	35	1498	29	28	28	2/15
1:f-auc	29	1726	11	11	10	5/15
f_{20}	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:auc	14	9	6	6	6	9/15
1:f-auc	7	7	5	5	5	10/15
f_{21}	41	1674	1705	1729	1757	14/15
0:auc	3	151	149	147	145	10/15
1:f-auc	5	110	109	107	106	11/15
f_{22}	71	938	1008	1040	1068	14/15
0:auc	6	469	437	424	414	8/15
1:f-auc	5	803	747	724	706	6/15
f_{23}	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:auc	2	2	3	5	7	15/15
1:f-auc	2	2	3	5	7	15/15
f_{24}	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:auc	5	0.5	0.3	0.3	0.3	2/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f_1	43	43	43	43	43	15/15
0:auc	100	275	449	625	799	15/15
1:f-auc	93	265	431	597	763	15/15
f_2	385	387	390	391	393	15/15
0:auc	51	72	91	110	129	15/15
1:f-auc	48	68	86	105	123	15/15
f_3	5066	7635	7643	7646	7651	15/15
0:auc	789	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_4	4722	7666	7700	7758	1.4e5	9/15
0:auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_5	41	41	41	41	41	15/15
0:auc	47	59	59	59	59	15/15
1:f-auc	42	53	54	54	54	15/15
f_6	1296	3413	5220	6728	8409	15/15
0:auc	21	15	15	15	15	15/15
1:f-auc	19	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	**14**$^{\star 3}$	15/15
f_7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:auc	6	2	2	2	2	15/15
1:f-auc	**5***	2	2	2	2	15/15
f_8	2039	4040	4219	4371	4484	15/15
0:auc	24	22	24	25	26	15/15
1:f-auc	23	23	24	25	26	15/15
f_9	1716	3277	3455	3594	3727	15/15
0:auc	26	26	28	29	30	15/15
1:f-auc	26	27	29	30	31	15/15
f_{10}	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:auc	3	3	2	3	3	15/15
1:f-auc	2	**2***	**2***	2	3	15/15
f_{11}	1002	6278	9762	1.2e4	1.5e4	15/15
0:auc	8	3	2	3	3	15/15
1:f-auc	**7***	2	2	2	2	15/15
f_{12}	1042	2740	4140	1.2e4	1.4e4	15/15
0:auc	27	18	19	9	10	15/15
1:f-auc	27	19	20	9	10	15/15
f_{13}	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:auc	26	12	2	3	3	15/15
1:f-auc	25	11	2	2	3	15/15
f_{14}	75	304	932	1648	1.6e4	15/15
0:auc	42	40	23	19	3	15/15
1:f-auc	33	38	23	19	3	15/15
f_{15}	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:auc	481	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f_{16}	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f_{17}	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:auc	25	7	7	4	5	12/15
1:f-auc	23	7	2	2	5	13/15
f_{18}	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:auc	12	2	3	5	6	11/15
1:f-auc	11	2	3	11	28	5/15
f_{19}	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:auc	1379	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
f_{20}	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:auc	48	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40***	∞	∞	∞	∞ *2.0e6*	0/15
f_{21}	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:auc	266	285	274	258	229	5/15
1:f-auc	12	568	547	515	456	3/15
f_{22}	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:auc	1086	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f_{23}	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:auc	1	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
f_{24}	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 13: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:auc is AUC-Bandit and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.11 *F-AUC-Bandit* versus F-SR-Bandit

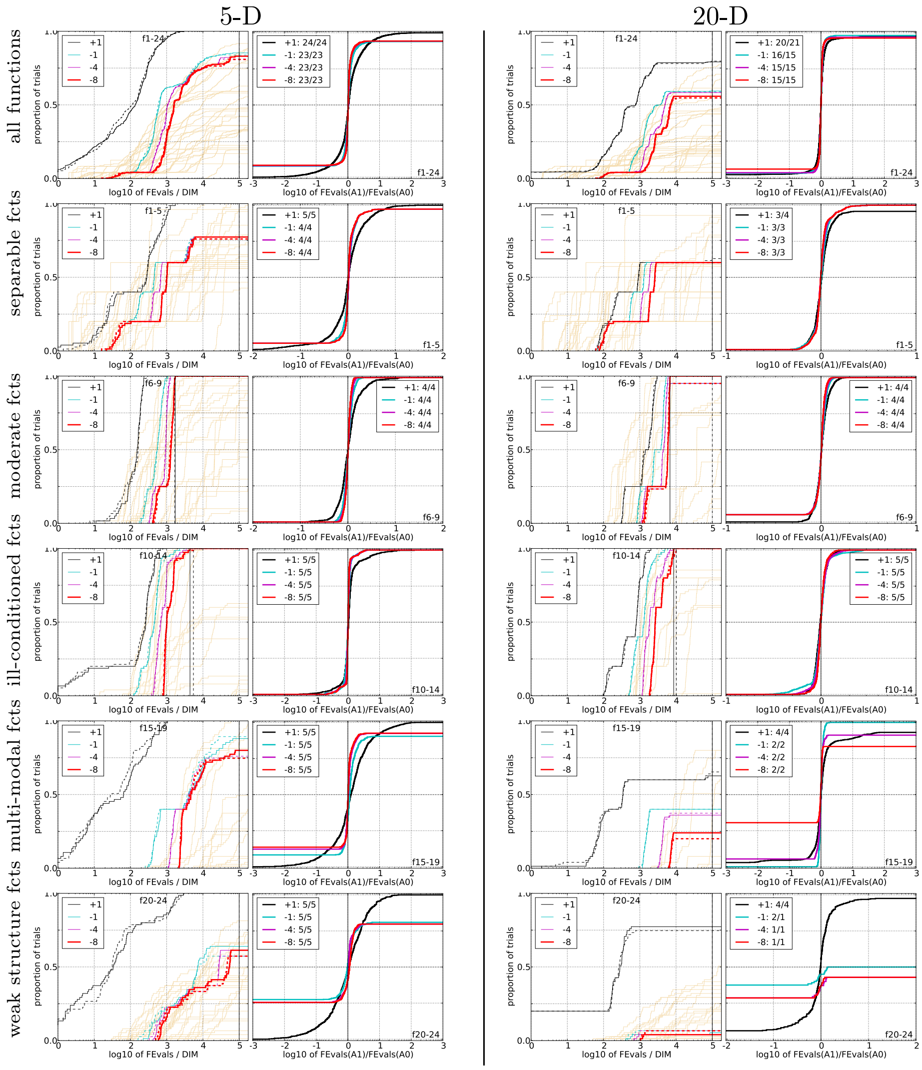

Figure 14: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and F-SR-Bandit (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by F-SR-Bandit, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	15/15
0:f-sr	6	69	129	197	262	15/15
1:f-auc	6	67	132	203	266	15/15
f2	83	88	90	92	94	15/15
0:f-sr	18	25	33	42	49	15/15
1:f-auc	18	26	35	42	50	15/15
f3	716	1637	1646	1650	1654	15/15
0:f-sr	4	87	88	88	88	12/15
1:f-auc	3	59	60	60	60	13/15
f4	809	1688	1817	1886	1903	15/15
0:f-sr	4	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f5	10	10	10	10	10	15/15
0:f-sr	12	20	20	20	20	15/15
1:f-auc	15	24	24	24	24	15/15
f6	114	281	580	1038	1332	15/15
0:f-sr	7	9	7	5	5	15/15
1:f-auc	6	8	7	5	5	15/15
f7	24	1171	1572	1572	1597	15/15
0:f-sr	10	1	1	1	1	15/15
1:f-auc	13	1	1	1	1	15/15
f8	73	336	391	410	422	15/15
0:f-sr	13	10	11	13	14	15/15
1:f-auc	13	11	11	13	14	15/15
f9	35	214	300	335	369	15/15
0:f-sr	25	17	15	16	17	15/15
1:f-auc	24	16	16	16	17	15/15
f10	349	574	626	829	880	15/15
0:f-sr	4	4	5	5	5	15/15
1:f-auc	4	4	5	5	5	15/15
f11	143	763	1177	1467	1673	15/15
0:f-sr	5	2	2	2	2	15/15
1:f-auc	6	2	2	2	2	15/15
f12	108	371	461	1303	1494	15/15
0:f-sr	23	14	15	7	7	15/15
1:f-auc	22	13	14	6	7	15/15
f13	132	250	1310	1752	2255	15/15
0:f-sr	10	11	3	3	3	15/15
1:f-auc	10	11	3	3	3	15/15
f14	10	58	139	251	476	15/15
0:f-sr	1	15	14	11	8	15/15
1:f-auc	2	15	13	11	8	15/15
f15	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:f-sr	4	7	7	7	7	12/15
1:f-auc	5	7	7	7	7	12/15
f16	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:f-sr	3	25	7	10	9	13/15
1:f-auc	6	31	12	11	10	13/15
f17	5	899	3669	6351	7934	15/15
0:f-sr	5	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
f18	103	3968	9280	1.1e4	1.2e4	15/15
0:f-sr	4	0.7	0.7	0.9	1.0	15/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
f19	1	242	1.2e5	1.2e5	1.2e5	15/15
0:f-sr	37	1120	60	59	59	1/15
1:f-auc	29	1726	11	11	10	5/15
f20	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:f-sr	9	15	11	11	11	7/15
1:f-auc	7	7	5	5	5	10/15
f21	41	1674	1705	1729	1757	14/15
0:f-sr	4	110	108	107	106	11/15
1:f-auc	5	110	109	107	106	11/15
f22	71	938	1008	1040	1068	14/15
0:f-sr	4	612	569	552	539	7/15
1:f-auc	5	803	747	724	706	6/15
f23	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:f-sr	1	2	3	5	**6***	15/15
1:f-auc	2	2	3	5	7	15/15
f24	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:f-sr	5	0.3	0.2	0.2	0.2	3/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	15/15
0:f-sr	90	259	430	604	766	15/15
1:f-auc	93	265	431	597	763	15/15
f2	385	387	390	391	393	15/15
0:f-sr	48	67	86	106	123	15/15
1:f-auc	48	68	86	105	123	15/15
f3	5066	7635	7643	7646	7651	15/15
0:f-sr	2872	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f4	4722	7666	7700	7758	1.4e5	9/15
0:f-sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f5	41	41	41	41	41	15/15
0:f-sr	41	51	51	51	51	15/15
1:f-auc	42	53	54	54	54	15/15
f6	1296	3413	5220	6728	8409	15/15
0:f-sr	18	13	13	14	14	15/15
1:f-auc	19	14	14	14	14	15/15
f7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:f-sr	5	17	10	10	10	14/15
1:f-auc	5	2	2	2	2	15/15
f8	2039	4040	4219	4371	4484	15/15
0:f-sr	22	97	96	94	94	13/15
1:f-auc	23	23	24	25	26	15/15
f9	1716	3277	3455	3594	3727	15/15
0:f-sr	26	27	29	30	31	15/15
1:f-auc	26	27	29	30	31	15/15
f10	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:f-sr	2	2	2	2	3	15/15
1:f-auc	2	2	2	2	3	15/15
f11	1002	6278	9762	1.2e4	1.5e4	15/15
0:f-sr	7	2	2	2	3	15/15
1:f-auc	7	2	2	2	2	15/15
f12	1042	2740	4140	1.2e4	1.4e4	15/15
0:f-sr	27	21	21	10	10	15/15
1:f-auc	27	19	20	9	10	15/15
f13	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:f-sr	25	11	2	2	2	15/15
1:f-auc	25	11	2	2	3	15/15
f14	75	304	932	1648	1.6e4	15/15
0:f-sr	34	37	22	19	3	15/15
1:f-auc	33	38	23	19	3	15/15
f15	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:f-sr	229	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f16	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:f-sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f17	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:f-sr	19	6	2	4	8	12/15
1:f-auc	23	7	2	2	5	13/15
f18	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:f-sr	11	2	3	9	56	3/15
1:f-auc	11	2	3	11	28	5/15
f19	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:f-sr	1347	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
f20	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:f-sr	42	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	40	∞	∞	∞	∞ *2.0e6*	0/15
f21	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:f-sr	11	284	274	258	229	5/15
1:f-auc	12	568	547	515	456	3/15
f22	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:f-sr	1574	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f23	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:f-sr	2	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
f24	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:f-sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 14: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:f-sr is F-SR-Bandit and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.12 ***F-AUC-Bandit*** versus SR-Bandit

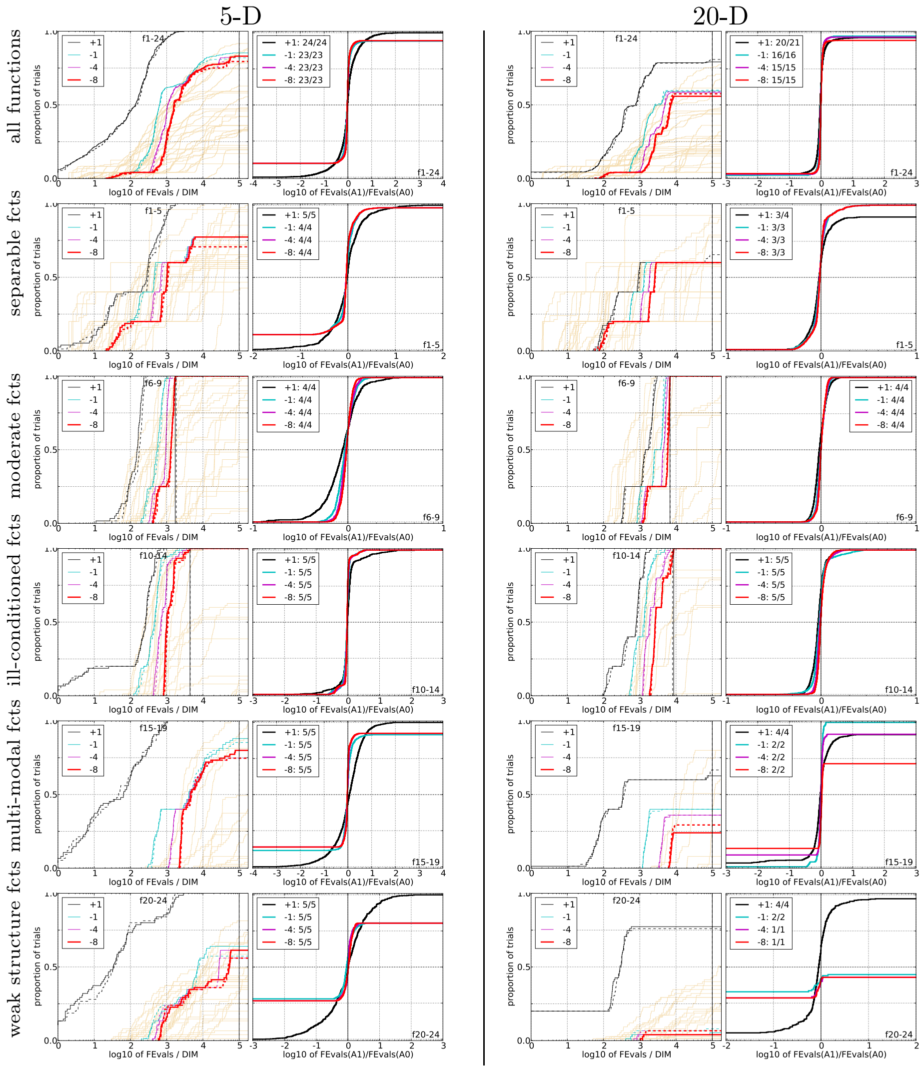

Figure 15: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and SR-Bandit (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by SR-Bandit, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	15/15
0:sr	5	74	139	213	282	15/15
1:f-auc	6	67	132	203	266	15/15
f2	83	88	90	92	94	15/15
0:sr	19	27	37	46	53	15/15
1:f-auc	18	26	35	**42***	50	15/15
f3	716	1637	1646	1650	1654	15/15
0:sr	4	278	277	277	277	8/15
1:f-auc	3	59	60	60	60	13/15
f4	809	1688	1817	1886	1903	15/15
0:sr	6	∞	∞	∞	∞ *5.0e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
f5	10	10	10	10	10	15/15
0:sr	17	28	28	28	28	15/15
1:f-auc	15	24	24	24	24	15/15
f6	114	281	580	1038	1332	15/15
0:sr	8	9	7	6	6	15/15
1:f-auc	6	8	7	5	5	15/15
f7	24	1171	1572	1572	1597	15/15
0:sr	15	1	1	1	1	15/15
1:f-auc	13	1	1	1	1	15/15
f8	73	336	391	410	422	15/15
0:sr	14	11	12	13	15	15/15
1:f-auc	13	11	11	13	14	15/15
f9	35	214	300	335	369	15/15
0:sr	29	17	15	16	17	15/15
1:f-auc	24	16	16	16	17	15/15
f10	349	574	626	829	880	15/15
0:sr	5	4	5	5	6	15/15
1:f-auc	4	4	5	5	5	15/15
f11	143	763	1177	1467	1673	15/15
0:sr	6	2	2	2	3	15/15
1:f-auc	6	2	2	2	2	15/15
f12	108	371	461	1303	1494	15/15
0:sr	26	15	16	7	7	15/15
1:f-auc	22	13	14	6	7	15/15
f13	132	250	1310	1752	2255	15/15
0:sr	10	11	3	3	3	15/15
1:f-auc	10	11	3	3	3	15/15
f14	10	58	139	251	476	15/15
0:sr	2	17	15	12	9	15/15
1:f-auc	2	15	13	11	**8***	15/15
f15	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:sr	5	5	5	5	5	13/15
1:f-auc	5	7	7	7	7	12/15
f16	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:sr	8	45	17	15	14	12/15
1:f-auc	6	31	12	11	10	13/15
f17	5	899	3669	6351	7934	15/15
0:sr	5	2	1	1	1	15/15
1:f-auc	6	2	1	1	1	15/15
f18	103	3968	9280	1.1e4	1.2e4	15/15
0:sr	3	0.7	0.7	0.9	1	15/15
1:f-auc	4	0.8	0.7	0.9	1.0	15/15
f19	1	242	1.2e5	1.2e5	1.2e5	15/15
0:sr	31	2488	60	60	59	1/15
1:f-auc	29	1726	11	11	10	5/15
f20	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:sr	10	12	8	8	8	8/15
1:f-auc	7	7	5	5	5	10/15
f21	41	1674	1705	1729	1757	14/15
0:sr	5	200	197	195	192	9/15
1:f-auc	5	110	109	107	106	11/15
f22	71	938	1008	1040	1068	14/15
0:sr	4	358	334	324	316	9/15
1:f-auc	5	803	747	724	706	6/15
f23	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:sr	2	2	3	5	6	15/15
1:f-auc	2	2	3	5	7	15/15
f24	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:sr	4	0.5	0.7	0.5	0.5	1/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	15/15
0:sr	99	269	436	607	778	15/15
1:f-auc	93	265	431	597	763	15/15
f2	385	387	390	391	393	15/15
0:sr	50	70	89	108	126	15/15
1:f-auc	48	68	86	105	123	15/15
f3	5066	7635	7643	7646	7651	15/15
0:sr	1372	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f4	4722	7666	7700	7758	1.4e5	9/15
0:sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f5	41	41	41	41	41	15/15
0:sr	45	56	56	56	56	15/15
1:f-auc	42	53	54	54	54	15/15
f6	1296	3413	5220	6728	8409	15/15
0:sr	20	14	14	14	14	15/15
1:f-auc	19	14	14	14	14	15/15
f7	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:sr	6	2	2	2	2	15/15
1:f-auc	5	2	2	2	2	15/15
f8	2039	4040	4219	4371	4484	15/15
0:sr	22	21	23	24	25	15/15
1:f-auc	23	23	24	25	26	15/15
f9	1716	3277	3455	3594	3727	15/15
0:sr	26	26	28	29	30	15/15
1:f-auc	26	27	29	30	31	15/15
f10	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:sr	3	3	2	3	3	15/15
1:f-auc	2	2	2	2	3	15/15
f11	1002	6278	9762	1.2e4	1.5e4	15/15
0:sr	8	3	2	2	3	15/15
1:f-auc	7	2	2	2	2	15/15
f12	1042	2740	4140	1.2e4	1.4e4	15/15
0:sr	27	17	18	9	9	15/15
1:f-auc	27	19	20	9	10	15/15
f13	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:sr	26	11	2	2	2	15/15
1:f-auc	25	11	2	2	3	15/15
f14	75	304	932	1648	1.6e4	15/15
0:sr	37	40	23	20	3	15/15
1:f-auc	33	38	23	19	3	15/15
f15	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:sr	185	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
f16	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
f17	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:sr	21	7	2	2	3	14/15
1:f-auc	23	7	2	2	5	13/15
f18	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:sr	12	2	8	11	13	8/15
1:f-auc	11	2	3	11	28	5/15
f19	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:sr	1472	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
f20	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:sr	48	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	**40***	∞	∞	∞	∞ *2.0e6*	0/15
f21	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:sr	12	285	274	258	229	5/15
1:f-auc	12	568	547	515	456	3/15
f22	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:sr	1086	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
f23	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:sr	2	433	∞	∞	∞ *2.0e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
f24	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:sr	∞	∞	∞	∞	∞ *2.0e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 15: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:sr is SR-Bandit and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

6.13 *F-AUC-Bandit* versus IPOP-CMA-ES

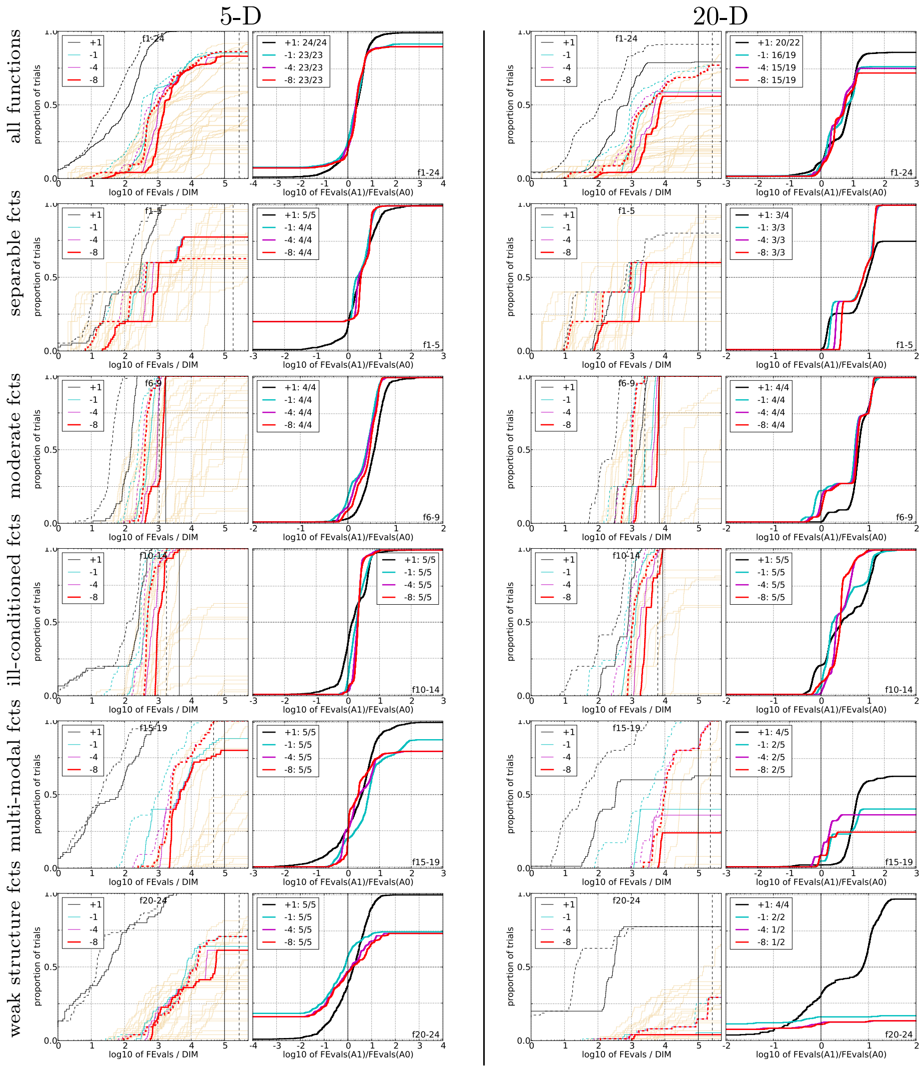

Figure 16: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for F-AUC (solid) and IPOP-CMA-ES (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of F-AUC divided by IPOP-CMA-ES, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (F-AUC first).

5-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	11	12	12	12	12	15/15
0:cma	2	**14**$^{\star 3}$	**27**$^{\star 3}$	**39**$^{\star 3}$	**51**$^{\star 3}$	15/15
1:f-auc	6	67	132	203	266	15/15
$\mathbf{f_2}$	83	88	90	92	94	15/15
0:cma	**14***	**18**$^{\star 3}$	**19**$^{\star 3}$	**21**$^{\star 3}$	**22**$^{\star 3}$	15/15
1:f-auc	18	26	35	42	50	15/15
$\mathbf{f_3}$	716	1637	1646	1650	1654	15/15
0:cma	2	3130	3113	3106	3099	2/15
1:f-auc	3	59	60	60	60	13/15
$\mathbf{f_4}$	809	1688	1817	1886	1903	15/15
0:cma	**2***	∞	∞	∞	∞ *8.5e5*	0/15
1:f-auc	6	∞	∞	∞	∞ *5.0e5*	0/15
$\mathbf{f_5}$	10	10	10	10	10	15/15
0:cma	**5**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	15/15
1:f-auc	15	24	24	24	24	15/15
$\mathbf{f_6}$	114	281	580	1038	1332	15/15
0:cma	**2**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	6	8	7	5	5	15/15
$\mathbf{f_7}$	24	1171	1572	1572	1597	15/15
0:cma	**4**$^{\star 2}$	1	1	1	1	15/15
1:f-auc	13	1	1	1	1	15/15
$\mathbf{f_8}$	73	336	391	410	422	15/15
0:cma	**3**$^{\star 3}$	**5**$^{\star 2}$	**6**$^{\star 2}$	**6**$^{\star 3}$	**6**$^{\star 3}$	15/15
1:f-auc	13	11	11	13	14	15/15
$\mathbf{f_9}$	35	214	300	335	369	15/15
0:cma	**6**$^{\star 3}$	**9***	**7**$^{\star 3}$	**7**$^{\star 3}$	**7**$^{\star 3}$	15/15
1:f-auc	24	16	16	16	17	15/15
$\mathbf{f_{10}}$	349	574	626	829	880	15/15
0:cma	4	**3**$^{\star 3}$	**3**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
1:f-auc	4	4	5	5	5	15/15
$\mathbf{f_{11}}$	143	763	1177	1467	1673	15/15
0:cma	9	2	**2**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	**6**$^{\star 2}$	2	2	2	2	15/15
$\mathbf{f_{12}}$	108	371	461	1303	1494	15/15
0:cma	**9**$^{\star 2}$	**6***	**6***	**3***	**3***	15/15
1:f-auc	22	13	14	6	7	15/15
$\mathbf{f_{13}}$	132	250	1310	1752	2255	15/15
0:cma	**3**$^{\star 3}$	**5**$^{\star 3}$	**1**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 2}$	15/15
1:f-auc	10	11	3	3	3	15/15
$\mathbf{f_{14}}$	10	58	139	251	476	15/15
0:cma	2	**4**$^{\star 3}$	**5**$^{\star 3}$	**5**$^{\star 3}$	**4**$^{\star 3}$	15/15
1:f-auc	2	15	13	11	8	15/15
$\mathbf{f_{15}}$	511	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0:cma	2	1	1	1	1	15/15
1:f-auc	5	7	7	7	7	12/15
$\mathbf{f_{16}}$	120	2662	1.0e4	1.2e4	1.2e4	15/15
0:cma	3	**2**$^{\star 3}$	**1.0**$^{\star 3}$	**0.9**$^{\star 3}$	**0.9**$^{\star 3}$	15/15
1:f-auc	6	31	12	11	10	13/15
$\mathbf{f_{17}}$	5	899	3669	6351	7934	15/15
0:cma	5	**1.0***	**0.8***	0.8	1	15/15
1:f-auc	6	2	1	1	1	15/15
$\mathbf{f_{18}}$	103	3968	9280	1.1e4	1.2e4	15/15
0:cma	**1**$^{\star 3}$	0.9	1	1	1.0	15/15
1:f-auc	4	0.8	0.7	**0.9***	1.0	15/15
$\mathbf{f_{19}}$	1	242	1.2e5	1.2e5	1.2e5	15/15
0:cma	21	**125**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	29	1726	11	11	10	5/15
$\mathbf{f_{20}}$	16	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0:cma	4	1	1	1	1	15/15
1:f-auc	7	7	5	5	5	10/15
$\mathbf{f_{21}}$	41	1674	1705	1729	1757	14/15
0:cma	6	30	31	31	31	14/15
1:f-auc	5	110	109	107	106	11/15
$\mathbf{f_{22}}$	71	938	1008	1040	1068	14/15
0:cma	12	166	161	158	155	11/15
1:f-auc	5	803	747	724	706	6/15
$\mathbf{f_{23}}$	3	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0:cma	2	33	15	14	14	11/15
1:f-auc	2	2	3	5	7	15/15
$\mathbf{f_{24}}$	1622	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0:cma	3	1	0.9	0.7	0.7	2/15
1:f-auc	4	0.1	0.2	0.1	0.1	4/15

20-D

Δf	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	43	43	43	43	43	15/15
0:cma	**8**$^{\star 3}$	**20**$^{\star 3}$	**33**$^{\star 3}$	**46**$^{\star 3}$	**58**$^{\star 3}$	15/15
1:f-auc	93	265	431	597	763	15/15
$\mathbf{f_2}$	385	387	390	391	393	15/15
0:cma	**35**$^{\star 3}$	**43**$^{\star 3}$	**45**$^{\star 3}$	**47**$^{\star 3}$	**48**$^{\star 3}$	15/15
1:f-auc	48	68	86	105	123	15/15
$\mathbf{f_3}$	5066	7635	7643	7646	7651	15/15
0:cma	**13**$^{\star 3}$	∞	∞	∞	∞ *2.9e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_4}$	4722	7666	7700	7758	1.4e5	9/15
0:cma	∞	∞	∞	∞	∞ *2.8e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_5}$	41	41	41	41	41	15/15
0:cma	**6**$^{\star 3}$	**7**$^{\star 3}$	**7**$^{\star 3}$	**7**$^{\star 3}$	**7**$^{\star 3}$	15/15
1:f-auc	42	53	54	54	54	15/15
$\mathbf{f_6}$	1296	3413	5220	6728	8409	15/15
0:cma	**2**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	19	14	14	14	14	15/15
$\mathbf{f_7}$	1351	9503	1.7e4	1.7e4	1.7e4	15/15
0:cma	**2**$^{\star 3}$	3	2	2	2	15/15
1:f-auc	5	**2***	2	2	2	15/15
$\mathbf{f_8}$	2039	4040	4219	4371	4484	15/15
0:cma	**4**$^{\star 3}$	**4**$^{\star 3}$	**4**$^{\star 3}$	**4**$^{\star 3}$	**4**$^{\star 3}$	15/15
1:f-auc	23	23	24	25	26	15/15
$\mathbf{f_9}$	1716	3277	3455	3594	3727	15/15
0:cma	**5**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	**6**$^{\star 3}$	15/15
1:f-auc	26	27	29	30	31	15/15
$\mathbf{f_{10}}$	7413	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0:cma	**2**$^{\star 3}$	**2**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	2	2	2	2	3	15/15
$\mathbf{f_{11}}$	1002	6278	9762	1.2e4	1.5e4	15/15
0:cma	11	**2**$^{\star 2}$	**1**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	**7**$^{\star 3}$	2	2	2	2	15/15
$\mathbf{f_{12}}$	1042	2740	4140	1.2e4	1.4e4	15/15
0:cma	**5**$^{\star 3}$	**6**$^{\star 3}$	**5**$^{\star 3}$	**2**$^{\star 3}$	**2**$^{\star 3}$	15/15
1:f-auc	27	19	20	9	10	15/15
$\mathbf{f_{13}}$	652	2751	1.9e4	2.4e4	3.0e4	15/15
0:cma	**7**$^{\star 3}$	6	**1**$^{\star 2}$	**2**$^{\star 2}$	2	15/15
1:f-auc	25	11	2	2	3	15/15
$\mathbf{f_{14}}$	75	304	932	1648	1.6e4	15/15
0:cma	**4**$^{\star 3}$	**4**$^{\star 3}$	**4**$^{\star 3}$	**6**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	33	38	23	19	3	15/15
$\mathbf{f_{15}}$	3.0e4	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0:cma	**1**$^{\star 3}$	**0.7**$^{\star 3}$	**0.7**$^{\star 3}$	**0.5**$^{\star 3}$	**0.5**$^{\star 3}$	15/15
1:f-auc	474	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{16}}$	1384	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0:cma	**2**$^{\star 3}$	**0.9**$^{\star 3}$	**0.8**$^{\star 3}$	**1**$^{\star 3}$	**1**$^{\star 3}$	15/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{17}}$	63	4005	3.1e4	5.6e4	8.0e4	15/15
0:cma	**2**$^{\star 2}$	**1**$^{\star 3}$	**0.8**$^{\star 3}$	**1.0**$^{\star 3}$	**1***	15/15
1:f-auc	23	7	2	2	5	13/15
$\mathbf{f_{18}}$	621	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0:cma	**1**$^{\star 3}$	1	1.0	1	1	15/15
1:f-auc	11	2	3	11	28	5/15
$\mathbf{f_{19}}$	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0:cma	**161**$^{\star 3}$	**0.7**$^{\star 3}$	**0.4**$^{\star 3}$	**0.4**$^{\star 3}$	**0.4**$^{\star 3}$	15/15
1:f-auc	1315	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{20}}$	82	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0:cma	**5**$^{\star 3}$	**0.6**$^{\star 3}$	**0.6**$^{\star 3}$	**0.6**$^{\star 3}$	**0.6**$^{\star 3}$	15/15
1:f-auc	40	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{21}}$	561	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0:cma	**4**$^{\star 2}$	110	106	100	88	7/15
1:f-auc	12	568	547	515	456	3/15
$\mathbf{f_{22}}$	467	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0:cma	445	∞	∞	∞	∞ *1.3e6*	0/15
1:f-auc	675	∞	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{23}}$	3	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0:cma	4	∞	∞	∞	∞ *2.5e6*	0/15
1:f-auc	2	440	∞	∞	∞ *2.0e6*	0/15
$\mathbf{f_{24}}$	1.3e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0:cma	∞	∞	∞	∞	∞ *5.1e6*	0/15
1:f-auc	∞	∞	∞	∞	∞ *2.0e6*	0/15

Table 16: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. 0:cma is IPOP-CMA-ES and 1:f-auc is F-AUC. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

7 Conclusions

This report presented an extensive empirical analysis of *Adaptive Strategy Selection* (*AdapSS*) techniques applied to the selection of mutation strategies within the Differential Evolution algorithm on continuous optimization problems. The results focused mainly on the *F-AUC-Bandit*, a comparison-based technique recently proposed [13] in the context of Genetic Algorithms, which was compared with: (i) the common naïve choices, *i.e.*, the use of a single strategy or the uniform strategy selection between the set of available ones; (ii) other previously proposed *AdapSS* techniques, namely, *PM-AdapSS-DE* [16], *AP* [28], and *DMAB* [7], the two latter being coupled with the extreme value based credit assignment [10]; (iii) the other rank-based approaches proposed in the same paper than *F-AUC-Bandit* (AUC, SR, F-SR) [13]; and (iv) the state-of-the-art continuous optimizer CMA-ES [3].

F-AUC-Bandit obtained significantly better results w.r.t. the naïve choices and the existent adaptive techniques in most of the functions; while almost no significant differences were found between it and the other rank-based approaches. Although showing a significant performance gain w.r.t. the base technique, the Differential Evolution, there is still a lot of space for improvements in order to turn it competitive w.r.t. state-of-the-art continuous optimizers such as the CMA-ES. Better performances can be achieved by tuning the DE parameters, what can also be done on-line, while solving the problem, as proposed for the SaDE algorithm in [25]. This is a possible path for future work.

Besides, none of the DE-based techniques was able to perform well on the multi-modal and weak-structure (which are also multi-modal) function classes. This might be related to the fact that just the fitness improvements are used to calculate the rewards assigned to the strategies, while in multi-modal problems the diversity should also be considered. Because of this, an analysis should be done concerning credit assignments that aggregate both measures, such as the ones proposed in [21].

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